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Studies on Hydromagnetic Stability of Newtonian and Non-Newtonian Fluid

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STUDIES ON HYDROMAGNETIC STABILITY OF NEWTONIAN AND NON-NEWTONIAN FLUIDS

THESIS SUBMITTED FOR THE DEGREE OF
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Certified that the thesis entitled "**Studies on Hydromagnetic Stability of Newtonian and Non-Newtonian Fluid.**" submitted by Mr. **M. Monsur Rahman** in fulfilment of the requirements for the degree of Doctor of Philosophy in Mathematics, University of Rajshahi, Rajshahi, has been completed under my supervision. I believe that this research work is an original one and it has not been submitted elsewhere for any degree.

(Dr. M. Shamsul Alam Sarker)
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ABSTRACT

In this thesis hydromagnetic stabilities with respect to both axisymmetric and non-axisymmetric perturbations of an incompressible perfectly conducting viscous fluid in presence of a magnetic field has been investigated. In chapter two, we have discussed the hydromagnetic stability with respect to axisymmetric disturbance of an in-compressible viscous fluid rotating between two perfectly conducting infinite co-axial cylinders in presence of a magnetic field by inner product method. The complex characteristic equation for the growth rate has been simplified and deduce propagation conditions for unstable, oscillatory and stable modes. In chapter three, hydromagnetic stability of helical flows in viscous fluid has been discussed. In chapter four, we have studied the flow of a highly conducting viscous incompressible fluid which is flowing between two paralled non-conducting planes in a uniform transverse magnetic field perpendicular to the plane. The result obtained have been discussed with the help of tables and graphs. In chapter five, we have investigated the unsteady MHD flow of a visco-elastic Rivlin Ericksen fluid with transient pressure gradient through a uniform circular cylinder in a uniform transverse magnetic field. Here, the velocity profile of a fluid element has been calculated theoretically and graphically.

In chapter six, an attempt has been made to investigate the unsteady flow of an incompressible visco-elastic Rivlin-Ericksen fluid between two concentric cylinders with transient pressure gradient in a uniform transverse magnetic field. Here we have calculated the velocity profile of a fluid element theoretically and graphically.

In chapter seven, we have investigated the unsteady unidirectional flow of an in-compressible visco-elastic oldroyd type fluid between two concentric cylinders under the action of a magnetic field with time varying body forces. Here we have calculated velocity profile of a fluid element theoretically and graphically, we also have discussed the stability of the velocity profile. In chapter eight, Hyromagnetic Stability of visco-elastic oldroyd fluid with the time varying body force through a rectangular channel has been discussed.

The following research papers which are extracted from this thesis have either been published, accepted for publication or communicated in different journals.

1. Hydromagnetic stability of helical flows in-viscous fluid. (Published in the Journal of Bangladesh Mathematical Society, Dhaka, Vol-20, 2000.

2. Hydromagnetic stabilities of a rotating viscous incompressible fluid. (Accepted for publication the Jour. Rajshahi Univ. Studies Vol. 9, 2001

3. Unsteady flow of visco-elastic Rivlin-Ericksen fluid with transient pressure gradient through a uniform circular cylinder. (Communicated for Publication)

4. MHD Flow of a highly conducting viscous fluid between non conducting parallel walls. (Communicated for Publication)

5. Unsteady MHD flow of visco-elastic oldroyd fluid between two concentric cylinder. Published in the Journal of Bangladesh Mathematical Society, Dhaka, Vo. 21, 2001.

6. Unsteady MHD flow of visco-elastic incompressible fluid between two concentric cylinders with transient pressure gradient. (Communicated for Publication)

7. Unsteady MHD flow of visco-elastic oldroyd fluid with the time varying body force through a rectangular channel. Accepted for publication in the Bulletin of the Calcutta Mathematical Society, Vol-96, 2004.

CHAPTER - I

INTRODUCTION

1-1 FLUID

All materials exhibit deformation under the action of forces. If the deformation in the material increases continually without limit under the action of shearing forces, however small, the material is called a 'fluid'. This continuous deformation under the action of forces is manifested in the tendency of fluids to flow.

Fluids are usually classified as liquids and gases. A liquid has intermolecular forces which hold it together so that it possesses volume but no definite shape. The liquids have definite volume which changes slightly when subjected to external forces or temperature differences. Thus a liquid may not occupy whole of the space of the container. A gas, on the other hand, consists of molecules in motion which collide with each other tending to disperse it so that a gas has no definite volume or shape. The inter-molecular forces are extremely small in gases. A gas has no definite volume and occupies the whole of the space of the container.

Newtonian Fluids: Newton, while discussing the properties of fluids, remarked that in a simple rectilinear motion of a fluid two neighbouring fluid layers, one moving over the other with some relative velocity, will experience a tangential force proportional to the relative velocity between the two layers and inversely proportional to the distance between the layers. That is, if the two neighbouring fluid layers are moving with velocities u and u+du and are at a distance ∂y then the shearing stress

$$\tau = \mu \frac{\partial u}{\partial y}.$$

This is called Newtonian hypothesis and a fluid satisfying this hypothesis is called a Newtonian fluid, and the constant of proportionality μ is called its coefficient of viscosity. The constitutive equation for Newtonian fluids is

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} - \frac{2}{3}\mu e_{u}\delta_{ij}.$$

Non-Newtonian Fluids: The Newtonian hypothesis worked very well in explaining many physical phenomena in various branches of fluid dynamics. This tempts us to remark that most of fluids at least in ordinary situations behave like Newtonian fluids. But in the recent years, especially with the emergence of polymers, it has been found that there are fluids which show a distinct deviation from Newtonian hypothesis. Such fluids are called non-Newtonian fluids.

Most of the theories developed in the recent years are formulated purely on the theoretical bases.

The non-Newtonian fluids are broadly classified into the following three categories:

(i) purely viscous fluids
(ii) visco-plastic fluids
and (iii) visco-elastic fluids.

The constitutive equation for non-Newtonian fluids is

$$\tau_{ij} = -p\delta_{ij} + p_{ij},$$

where p_{ij} is the shearing stress tensor, p_{ij} is zero when the fluid is at rest, and $p_{ij} = 0$, if $e_{ij} = 0$.

The fluid in which the stress tensor p_{ij} is a given function of the strain rate is called a purely viscous fluid. Mathematically this can be stated as

$$p_{ij} = f(e_{ij}), f(0) = 0 .$$

A fluid satisfying the constitutive equation

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} + \mu_1 e_{ik} e_{kj} + \mu_2 \delta_{ij},$$

is called Reiner-Rivlin fluid where μ and μ_1 and are called the coefficient of viscosity and cross viscosity respectively. When μ, μ_1 and μ_2 depends on the invariants of e_{ij} the fluid is called generalised Reiner-Rivlin fluid. One of the important observations in the viscous fluids is that if we apply a certain shearing stress on a fluid, however small it may be, it causes a continuous deformation in the fluid. But in many materials like paints, pastes, etc, we find that if we apply a shearing stress less than a certain quantity, the material does not move at all. But when this shearing stress exceeds a certain value the material starts moving and the strain rate of the material depends upon the applied stress. Such materials are called plastics. Plastics behave like solids if the shearing stress is less than the critical shearing stress; and behave like a fluid if the shearing stress exceeds the critical shearing stress.

A new general empirical model of visco-elastic fluid has been suggested by P.R. Sengupta and S.K. Kundu in the following form

$$\tau_{ij} = -p\delta_{ij} + p_{ij}$$

$$(1 + \sum_{j=1}^{n} \lambda_j \frac{\delta^j}{\delta t^j}) p_{ij} = 2\mu (1 + \sum_{j+1}^{n} \mu_j \frac{\delta^j}{\delta t^j}) e_{ij}$$

$$e_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i}),$$

where τ_{ij} is the stress tensor, p_{ij} is the deviatory stress tensor, e_{ij} the rate of strain tensor, p is the fluid pressure, λ_j are new material constants of which the greatest value λ_1 represent the relaxation time parameter and $\lambda_2, \lambda_3, \ldots\ldots\ldots\ldots \lambda_n$ are additional material constants; μ_j are also new material constants of which the greatest value μ_1 represents the strain rate retardation time parameter and $\mu_2, \mu_3, \ldots\ldots\ldots\ldots \mu_n$ are additional constants representing the behaviour of a very wide class of visco-elastic liquids, δ_{ij} the metric tensor in Cartesian coordinates and μ, the coefficient of viscosity and v_i the velocity components. The material constant λ_j and μ_j designating visco-elasticity satisfy the following conditions

$$\lambda_1 > \lambda_2 > \lambda_3 \ldots\ldots\ldots\ldots\ldots\ldots > \lambda_n > 0$$

and

$$\mu_1 > \mu_2 > \mu_3 > \ldots\ldots\ldots\ldots\ldots\ldots > \mu_n > 0$$

i.e. they are arranged in descending order of magnitudes.

1-2 CONTINUUM HYPOTHESIS

In its most fundamental form, at the microscopic level, the description of the motion of a fluid involves a study of the behaviour of all the discrete molecules which make up the fluid. However, when one is dealing with problems in which some characteristic length in the flow is very large compared with molecular distances, it is convenient to think of a lump of fluid sufficiently small from macroscopic point of view but large enough at the microscopic level so as to contain a large number of molecules (for instance, at normal temperature and pressure a volume of 10^{-12}CC. of a gas contains about 2.7×10^{7} molecules) and to work with the average statistical properties of such large number of molecules. In such a case the detailed molecular structure is washed out completely and is replaced by a continuous model of matter having appropriate continuum properties so defined as to ensure that on the macroscopic scale the behaviour of the model resembles with the behaviour of the real large compared with molecular distances, the continuum model is invalid and the flow must be analysed on the molecular scale.

The smallest lump of fluid material having sufficiently large number of molecules to allow statistically of a continuum interpretation is here called a "fluid particle". The material in this thesis will deal primarily with fluids obeying continuum hypothesis.

1-3 VISCOSITY

Viscosity represents that property of an actual fluid which exhibits a certain resistance to alteration of form. Although this resistance is comparatively small for many practically important fluids, such as water or gases, it is not negligible. For other fluids, such as oil, glycerine etc., this resistance is quite large. In a viscous fluid, both tangential and normal forces exist. Some of the kinetic energy of flow will be dissipated as heat through the viscous forces.

We shall consider only the so-called Newtonian fluids, these representing most of the fluids encountered in ordinary engineering problems. The following discussion is applicable to such fluids.

Let the fluid be between two parallel plates separated by a distance y_o from each other. Let the lower plate be fixed, while the upper plate is moving uniformly with a velocity U and in a direction parallel to the lower one. A resistance D is experienced which is given by the formula,

$$D = A_0 \mu \frac{U}{y_0}, \qquad \text{........(3.01)}$$

where A_o is the area of the upper plate and μ is a constant of proportionality called the coefficient of viscosity.

It is an experimental fact that for an ordinary fluid the relative velocity at the solid surface is zero, i.e. there is no slip at the wall. The fluid is displaced in such a manner that the various layers of the fluid slide uniformly over one another, the velocity u of a layer of the fluid at a distance y from the lower plate is then

$$u = \frac{uy}{y_0}. \qquad \text{........(3.02)}$$

Experimental results show that the tangential force per unit area, or the shearing stress τ is proportional to the slope of the velocity, i.e.,

$$\tau = \mu \frac{du}{dy}. \qquad \text{.........(3.03)}$$

This linear relation is found to be very closely correct. The factor μ depends on the temperature T, but is independent of the pressure p for gasses at ordinary temperature. Discrepancies from the above law are observed only at very high velocities.

The dimension of the coefficient of viscosity μ are easily determined from equation (3.03)

$$\mu = \frac{\text{Shearing Stress}}{\text{Velocity gradient}} = \frac{mL/t^2L^2}{L/tL} = \frac{m}{tL}, \qquad \text{.........(3.04)}$$

where m is the mass, t is the time and L is the length.

From the simple kinetic theory of gasses, one may show that the coefficient of viscosity μ is proportional to the square root of the absolute temperature T. In actual fact the viscosity of a gas does rise with a rise of temperature, but the square-root variation of the simple kinetic theory of gases is only qualitatively correct. In practice we usually assume that the coefficient of viscosity is proportional to a power of the absolute temperature, i.e.,

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n. \qquad \text{.........(3.05)}$$

The equation (3.03) may be regarded as the definition of viscosity. Thus the coefficient of viscosity of fluid may be defined as the tangential force required per unit area to maintain a unit velocity gradient, i.e., to maintain unit relative velocity between two layers at unit distance apart.

For liquids the viscosity μ is nearly independent of pressure and decreases rapidly with increasing of temperature. In the case of gases, to a first approximation, the viscosity can be taken to be independent of pressure but it increases with increase of temperature.

The shearing stress of equation (3.03) is only one component of the stress tensor in the more general case. In a three dimensional flow, the stress tensor has the nine components

$$\left.\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{matrix}\right] , \qquad \text{........(3.06)}$$

where σ denotes the normal stress on the surfaces, i.e., the stress perpendicular to the surface considered. Hence σ_x is the normal stress on the surface perpendicular to the axis of x. The shearing stress is denoted by τ which is the stress in the surface considered. The first subscript refers to the direction of the axis perpendicular to the surface considered and the second subscript refers to the direction of the force in the surface. Thus τ_{xy} denotes the component of the shearing stress in the surface perpendicular to the x-axis in the direction of the y-axis. It can be shown that, for the six tangential stresses, those which have the same suffixes but in reversed order are equal.

This result follows from the condition of equilibrium of moments on an element in the continuum, i.e., $\tau_{xy} = \tau_{yx}$, etc.

For an ideal fluid, the tangential stresses are zero. By definition, the pressure is taken as the negative value of the normal stress. In an ideal fluid, we have

$$\begin{aligned} \sigma_x = \sigma_y = \sigma_z = -\rho \\ \tau_{xy} = \tau_{yz} = \tau_{zx} = 0 \end{aligned} \qquad \text{........(3.07)}$$

For inviscid fluid flow, the equations of motion are based on equation (3.07). It is easy to show that the x-, y-, and z- components of force per unit volume due to the non-homogeneous state stresses are respectively:

$$\left.\begin{aligned} X &= \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ Y &= \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \\ Z &= \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} \end{aligned}\right\} \qquad \text{........(3.08)}$$

For an inviscid fluid, equation (3.08) reduces simply to

$$X = -\frac{\partial p}{\partial x},\ Y = -\frac{\partial p}{\partial y},\ \text{and } Z = -\frac{\partial p}{\partial z}.$$

For a viscous fluid, we must express the viscous stresses in terms of the rate of change of velocity. In a fluid, there is a resistance to the time rate of change of shape, i.e., the deformation of velocity which may be called the strain in fluid flow. In three-dimensional flow, there are six quantities for the strain.

$$\left.\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y}, \\ \epsilon_z &= \frac{\partial \omega}{\partial z} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial \omega}{\partial y} \\ \gamma_{zx} &= \frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z} \end{aligned}\right], \qquad \text{........(3.09)}$$

where $\in$ is the normal strain, γ is the shearing strains, and u, v, w are the x', y', and z' components of velocity respectively. The strains also form a tensor of second order, namely,

$$\left.\begin{matrix} \in_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{2x}}{2} \\ \frac{\gamma_{xy}}{2} & \in_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{yz}}{2} & \in_z \end{matrix}\right\} \qquad \text{........(3.10)}$$

The sum of two of the quantities $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$......... etc., gives the Shearing Strain, where as the difference of two of these quantities gives the angular rotation of the fluid element. The components of rotation of a fluid element are

$$\left.\begin{aligned} \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \end{aligned}\right], \qquad \text{........(3.11)}$$

where ω_x, ω_y and ω_z are the average rate of rotation of the fluid element about the x', y' and z' axes, respectively. These angular rotations do not give internal stress but the strain does. The vortices is defined as twice the rate of rotation.

To a first approximation, the relations between stress and strain in a viscous fluid are

$$\left.\begin{aligned} \sigma_x &= -\underline{p} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu\frac{\partial u}{\partial x} \\ \sigma_y &= -\underline{p} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu\frac{\partial v}{\partial y} \\ \sigma_z &= -\underline{p} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu\frac{\partial w}{\partial z} \end{aligned}\right\} \qquad \text{........(3.12)}$$

$$\tau_{xy} = \mu\gamma_{xy},\ \tau_{yz} = \mu\gamma_{yz},\ \tau_{zx} = \mu\gamma_{zx},$$

where p is the hydrostatic pressure in a frictionless fluid, μ is the ordinary coefficient of viscosity, and λ is the second coefficient of viscosity. The only restrictions on the existence of two independent coefficient of viscosity are

$$\mu \geq 0 \qquad \text{........(3.13)}$$

$$2\mu + 3\lambda \geq 0 \qquad \text{........(3.14)}$$

We conclude that the relationship between the components of the viscous stress tensor and those of rate of strain tensor is given by

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{Kk}\delta_{ij}, \qquad \text{........(3.15)}$$

where $e_{ij} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}$ is a symmetric second order tensor, the diagonal elements of which represent rates of normal stains, while the non-diagonal terms represents rates of shearing strains. The tensor e_{ij} is, therefore, known as the rate strain tensor.

1-4 DYNAMIC SIMILARITY

We have developed the fundamental equations governing the flow of a viscous compressible fluid but there are no known general method to solve these equations. The main reason of the absence of such a general method is the nonlinear character of the governing equations. Only in few particular cases and that too under restricted conditions exact solutions of these equations for all ranges of viscosity, exist. However, attempts have been made to simplify these equations for two extreme case of viscosity, very large and very small, and we have well established theories for these cases which are known as 'Theory of slow motion' and 'Theory of boundary layers', but the cases of moderate viscosities can not be interpolated from these two theories. Even in these two extreme cases most of the research on the behaviour of viscous fluids has been carried out by experiments. In experiments, generally, a prototype (geometrically similar but reduced in size) of the actual body is taken and the flow around it is investigated in the wind tunnel in order to reduce the cost of the full scale test and to have better control over the conditions. This always raises the question of 'dynamic similarity' of fluid motions because, naturally, we would like to know that how far the results obtained on the prototype can be considered the same as on the full scale body.

Two fluid motions are said to be 'dynamically similar' if, with geometrically similar boundaries, the flow patterns are geometrically similar.

We now discuss the conditions under which the fluid motions are dynamically similar. In other words we have to find out those parameters which characterise a flow problem. There are two methods for finding out these parameters (i) inspection analysis, and (ii) dimensional analysis. In the inspection analysis, we reduce the fundamental equations to a non-dimensional form and obtain the non-dimensional parameters from the resulting equations. In dimensional analysis we form non-dimensional parameters from the physical quantities occurring in a problem even when the knowledge of the governing equations is missing. We will now discuss these with particular reference to the flow of a viscous compressible fluid.

The Navier-Stokes equation of motion of a viscous incompressible fluid in the x-direction is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad \text{........(4.01)}$$

Suppose L, V, P denote a characteristic length, velocity and pressure respectively. Thus writing

$$\left.\begin{aligned} x &= Lx' \\ y &= Ly' \\ z &= Lz' \end{aligned}\right\} \qquad \text{........(4.02)}$$

$$\left.\begin{aligned} u &= Vu' \\ v &= Vv' \\ w &= Vw' \end{aligned}\right\} \qquad \text{........(4.03)}$$

$$p = Pp', \qquad \text{........(4.04)}$$

where all primed quantities are pure numbers having no dimensions. Then, since L/V is the characteristic time,

$$\frac{\partial u}{\partial t} = \frac{\partial(Vu')}{\partial(L/V't)} = \frac{V^2}{L}\frac{\partial u'}{\partial t'}$$

$$u\frac{\partial u}{\partial x} = Vu'\frac{\partial(Vu')}{\partial(Lx')} = \frac{V^2}{L}u'\frac{\partial u'}{\partial x'} \text{ etc.}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{1}{\rho}\frac{\partial(Pp')}{\partial(Lx')} = \frac{P}{\rho L}\frac{\partial p'}{\partial x'}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2(Vu')}{\partial(Lx')^2} = \frac{V}{L^2}\frac{\partial^2 u'}{\partial x'^2}$$

Substituting these results into (4.01) and simplifying gives

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'} = \frac{LX}{V^2} - \frac{P}{\rho V^2}\frac{\partial P'}{\partial x'} + \frac{\nu}{VL}\nabla^2 u'. \qquad \text{........(4.05)}$$

In equation (4.05), the L.H.S. is entirely dimensionless. Hence the R.H.S. must likewise be so. It follows that the three quantities

$$\frac{LX}{V^2}, \frac{P}{\rho V^2}, \frac{\nu}{VL} \text{ must be dimensionless quantities.}$$

(i) Reynolds number R_e. This is the ratio of the inertia force to the viscous force

$$\therefore R_e = \frac{F_i}{F_v} = \frac{\rho L^2 V^2}{\mu VL} = \frac{LV}{\nu}.$$

The Reynolds number is important when the viscous force predominant. It is generally used to correlate meter coefficient, pipe friction coefficient and drag coefficient.

(ii) Froude number F_n. It is the ratio of the inertia force to the external gravity force.

$$\therefore F_n = \frac{\rho V^2 L^2}{\rho L^3 g} = \frac{V^2}{gL}.$$

The Froude number is important in open channel flow. it is useful in study of hydraulic pump, designing of hydraulic structures amd ships.

(iii)Eular number E_n. It is the ratio of the inertia force to the pressure force

$$\therefore E_n = \frac{\rho V^2 L^2}{\rho L^2} = \frac{\rho V^2}{P}.$$

The Euler number is important in the flow problems in which a pressure gradient exist.

(iv) Cauchy number C_n. It is the ratio of the inertia force to the elasticity force

$$\therefore C_n = \frac{\rho V^2 L^2}{kL^2} = \frac{\rho V^2}{K} = \frac{V^2}{(K/\rho)}.$$

The square root of the Cauchy number is the Mach number

$$\therefore M = \sqrt{C_n} = \frac{V^2}{(K/\rho)}.$$

The mach number is also defined as the ratio of the velocity V of the fluid to the velocity of sound a in that medium, i.e., $M = \frac{v}{a}$.

The Mach number is important in compressible fluid flow problems at high velocities, such as high velocity flow in pipes or motion of high speed projectiles and missiles.

(v) Weber number (W_n). It is the ratio of the inertia force to the surface tension force.

Thus

$$W_n = \frac{\rho V^2 L^2}{\sigma L} = \frac{\rho V^2 L}{\sigma}.$$

The Weber number is important for small jets of liquids, droplet formation and formation of waves.

Similitude may be summarise as follows.

i) Models are generally used to study complete flow phenomena which cannot be solved by mathematical analysis.

ii) For the model to yield useful information about the characteristic of the prototype, the model must have geometric, kinematics and dynamic similarity with the prototype.

iii) For complete dynamic similarity between the model and its prototype (a) the ratio of the inertia forces of the two systems must be equal to the ratio of the resultant forces, and (b) the ratio of inertia forces of the two systems must also be equal to the ratio of individual components.

iv) For complete dynamic similarity, the Euler, Reynolds, Froude, Mach and Weber numbers should be same in the model and prototype. It is impossible to satisfy all these requirements simultaneously in a model. Fortunately, in most fluid problems, only two or three types of forces are predominant. A particular state of fluid motion is usually simulated in a model by considering only the predominant forces.

v) In most fluid phenomena, the pressure force is taken as dependent variable as it depends on the motion being studied. Thus the Euler number will be automatically satisfied if the other relevant numbers are satisfied.

vi) A distorted model is one which is not completely similar to its prototype. Distorted models are generally used for rivers and open channels in order to get the flow characteristics identical to that in the prototype. Experience, judgement and sound knowledge of the fluid phenomenon are essential for proper interpretation of results from distorted models, as they are mainly qualitative and not quantitative.

vii) Models are provided with a movable bed in the cases where scour and deposition are to be simulated.

viii) As it is impossible to achieve simultaneous compliance of all similarity laws, some discrepancy in extrapolating results to the prototype occurs. This is known as scale effect.

A valuable means of detecting scale effect is to construct models of different scales and to compare the results.

1-5 DIMENSIONAL ANALYSIS

In dynamical similarity we reduced the governing equations of a viscous compressible fluid to a non-dimensional form and obtained the dimensional parameters. An alternative method, with which the non-dimensional parameters may be formed from the physical quantities occurring in flow problem is known as dimensional analysis. They are derived on the basis of the dimensions in which each of the quantities involved in a phenomenon is expressed, and hence, must not depend on the units chosen for the calculations. Dimensional analysis helps in obtaining a systematic form of the variables involved in a particular fluid phenomenon. It gives a sound and orderly arrangement of the variables involved in the problem. However, dimensional analysis does not give the complete relationship. It gives only a general expression. Investigations have to be done to obtain the complete expression. The numerical values of the coefficients are usually obtained from investigations.

In dynamics of viscous compressible fluids there are four fundamental units, viz., length, mass, time and temperature in which the dimensions of all the physical quantities occurring in such a flow problem can be expressed. We shall denote the dimensions of these fundamental units by [L], [M], [T] and [θ] respectively.

The methods of dimensional analysis are based on the Fourier's principle of dimensional homogeneity. The following two methods of dimensional analysis are commonly used.
(i) Rayleigh's method (ii) Buchingham's π method

Rayleigh's Mehod: In 1899, Lord Rayleigh proposed a method of dimensional analysis. He used this method for determining the effect of temperature on the viscosity of gases. In this method, the functional relationship is expressed in exponential form; for example, if Y is some function of independent variables X_1, X_2, X_3, ...etc, the functional relationship can be written as

$$Y = \varphi(X_1, X_2, X_3 \ldots\ldots\ldots\ldots)$$

$$= C_1\left(X_1^a, X_2^b, X_3^c \ldots\ldots\ldots\ldots\right), \qquad \ldots\ldots(5.01)$$

in which C_1 is a dimensionless coefficient which can be determined either from the physical characteristics of the problem or from experiments. Equation (5.01) being a physical equation, is dimensionally homogeneous. According to the principle of dimensional homogeneity, the exponents of the dimensions on both sides must be same. By equating the exponents on both sides, a set of simultaneous equations is obtained.

The exponents can be determined by solving these simultaneous equations. The Rayleigh method of dimensional analysis is difficult to use when a large number of variables are involved. The Buckingham π theorem may be used in such problems.

Buckingham's π theorem: The Buckingham π theorem states that if there are n variables in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such as, L, M, T) they may be grouped into (n-m) non-dimensional parameters. Buckingham called these non-dimensional parameters as π-terms. Each π term contains m primary variables, which are also called the repeating variables. The repeating variables appear in all π terms. In addition to these m repeating variables, each π-term contains one more variable of the remaining (n-m) variables. Thus, if X_1, X_2, and X_3 are taken as repeating variables,

then

$$
\begin{aligned}
\pi_1 &= X_1^{a_1} X_2^{b_1} X_3^{c_1} X_4 \\
\pi_2 &= X_1^{a_2} X_2^{b_2} X_3^{c_2} X_5 \\
&\vdots \\
\pi_{n-m} &= X_1^{a_{n-m}} X_2^{b_{n-m}} X_3^{c_{n-m}} X_n
\end{aligned}
$$

where indices a_1, b_1, c_1; a_2, b_2, c_2, etc., are constants to be determined as explained later.

In the dynamics of viscous compressible fluid the physical quantities involved are L, U, ρ, μ, K, g, P, C_p, T and the fundamental units in which the dimensions of all these quantities can be expressed are length, mass, time and temperature.

Let us take L, U, ρ and K as base quantities and

Let

$$
\begin{aligned}
\pi_1 &= L^{a_1} U^{b_1} \rho^{c_1} K^{d_1} \mu \\
\pi_2 &= L^{a_2} U^{b_2} \rho^{c_2} K^{d_2} g \\
\pi_3 &= L^{a_3} U^{b_3} \rho^{c_3} K^{d_3} P
\end{aligned}
$$

etc

Now $[\pi_1] = \left[(L)^{a_1}(LT^{-1})^{b_1}(L^{-3}M)^{c_1}(LMT^{-3}\theta^{-1})^{d_1}(L^{-1}MT^{-1})^{d_1}(L^{-1}MT^{-1})\right]$
$= \left[L^{a_1+b_1-3c_1+d_1-1}M^{c_1+d_1+1}T^{-b_1-3d_1-1}\theta^{-d_1}\right]$

If π_1 is dimensionless, then we must have

$$a_1 + b_1 + 3c_1 + d_1 - 1 = 0$$
$$c_1 + d_1 + 1 = 0$$
$$-b_1 - 3d_1 - 1 = 0$$
$$-d_1 = 0.$$

therefore, $a_1 = -1, b_1 = -1, c_1 = -1, d_1 = 0$.

Hence, $\pi_1 = L^{-1}U^{-1}\rho^{-1}\mu = \dfrac{\mu}{LU\rho} = \dfrac{1}{R_e}$

In a similar manner, we find that $\pi_2 = \dfrac{L_g}{U^2} = \dfrac{1}{F_n}, \pi_3 = \dfrac{P}{\rho U^2} = \dfrac{1}{E_n}$.

1-6 BASIC CONCEPTS OF STABILITY THEORY

In a theoretical discussion of any realistic flow we have many factors which we may not be able to account for. Even in any experiment of a fluid flow, there are some inherent disturbances which we may not be able to avoid. In order to obtain a laminar flow through a pipe physically we should know the reaction of the flow to such disturbances; that is, we are interested to know whether such disturbances decay or grow with time. If these disturbances decay with time, then we shall be able to realize that flow.

In any flow the disturbances contain some kinetic energy and if there is a transfer of energy from the disturbances into the basic flow then the magnitude of the disturbances will decrease and thus the flow will be stable. In a dissipative system a flow will be stable if the dissipative energy exceed the energy transferred to the disturbances from the main flow.

Mathematically, a system, whose stability is under some arbitrary perturbations and if these perturbations decay with time; i.e., the system returns to its original position. Then the system is said to be stable, otherwise the system is said to be unstable.

In linear stability theory we take the perturbations to be arbitrarily small and so we neglected those terms in the perturbations and their derivatives as compared to linear terms. Therefore in linear stability theory, the perturbations either grow exponentially or decay exponentially or the magnitude of the perturbations remain constant. If the perturbations decay exponentially then the system is said to be stable and if their magnitude remains constant then the system is said to be in the marginal state.

There are two main methods for analyzing the stability of any given flow.

i) The energy method

ii) The normal mode technique

In energy method we calculate the kinetic energy of the perturbations and if this kinetic energy decays with time then the flow is stable, otherwise unstable.

The normal mode technique is more widely used. This is because it is applicable to a wider class of problems. In this method, in linear theory, we assume that the perturbations are arbitrary small in magnitude so that the non-linear terms in the perturbations variables and their derivatives can be neglected as compared to the linear terms.

Moreover, we assume that the perturbations are regular functions of space variables and therefore the Fourier analysis of the perturbations is possible. Thus in this system the arbitrary perturbations are split into some fundamental modes and the reaction of the system to all such modes is observed. If the system is stable with respect to each mode, then the flow is stable, and if there is even one mode for which it is unstable then the flow is unstable, because after some time this mode will dominate over the whole flow.

1-7 MODES OF INSTABILITY

A number of types of instability have been studied in fluid mechanics. One is that met when laminar flow past rigid boundaries breaks down at large Reynolds numbers, and turbulence ensues. A second is the Kelvin-Helmholtz instability, responsible for the generation of ripples, when a wind blows over the surface of water at rest. A third is thermal instability leading to convection in a fluid heated from below:

A similar instability is important in the atmosphere. A fourth is the Richardson instability, induced in a thermally stable atmosphere by a vertical gradient in the (horizontal) wind velocity. All these, and others, have counter parts in MHD. Theoretical work on instability is usually based on normal-mode analysis. The basic state of equilibrium or steady motion is assumed to undergo a small perturbation involving a time factor $e^{-i\omega t}$, the possible values of ω being determined from the equations of motion and the boundary conditions. In stability can arise in either of two ways as the parameters of the problem are varied. The first is when ω^2 passes from positive to negative real values; this means that initial oscillatory motion is replaced by a perturbation which increases exponentially. The second is when ω is complex, and its real part passes from positive to negative values; this leads to a steadily increasing oscillation. The second case arises when the restoring forces during an oscillation push the material back towards the undisturbed state with a velocity greater than its original out word velocity, it is therefore sometime called over stability. It is encountered chiefly in problems involving dissipation or steady rotation

The normal mode method is not altogether adequate, particularly in discussing the stability of lamilar flow; a flow may be stable for small disturbances but unstable for large ones.

1-8 NATURE OF MAGNETO HYDRODYNAMICS

Magneto hydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluid across the magnetic field. When a conductor carrying an electric current moves in a magnetic field it experiences a force tending to move it at right angles to the electric field. Conversely, when a conductor moves in a magnetic field a current is induced in the magnetic field, a current is induced in the conductor in a direction mutually at right-angles to both the field and the direction of motion. These two statements, first enunciated by Faraday, constitute the laws of electromagnetism. The first is the principle of the electric motor: The second that of the dynamo. There is nothing in them to suggest that the conductor must be a solid. In fact suggestions have been made that the motion of the sea may produce perturbations in the earth's magnetic field. Further, tidal waves sweeping up the estuary of a river will cut the terrestrial lines of force and generate a current which can be detected in a cable connecting two electrodes placed in the river on opposite banks.

In the case when the conductor is either a liquid or a gas, electromagnetic forces will be generated which may be of the same order of magnitude as the hydrodynamical and inertial forces. Thus the equations of motion will have to take these electromagnetic forces into account as well as the other forces. The science which treats these phenomena is called magnetohydrodynamics (MHD). Other variants of nomencreature are: hydromagnetics, magneto-fluid dynamics, magneto-gas dynamics, etc.

Most liquids and gases are poor conductors of electricity. As a consequence their motion can normally be treated by the principles of fluid dynamics which have so far been studied in this thesis. However, it is possible to make some gases very highly conducting by ionizing them. For ionization to take effect, the gas must be very hot – at temperatures upwards of 5000^0K or so. Such ionized gases are called plasmas. The material within a star is a plasma of very high conductivity and it exists with in a strong magnetic field. Consequently we expect MHD effects to be realized in Star. Further, at the engineering level, experiments have been made for electric power generation by passing an ionized gas between the poles of a strong electromagnet so that an electric current would be generated at right angles to the magnetic field and to the direction of flow of the plasma, the current being collected by two spaced electrodes at right-angles to the direction of the current flow. At the present time MHD generators are not a practical possibility owing to the difficulties of producing suitably efficient and stable plasmas and sufficiently refractory materials to withstand the high temperatures of the plasmas.

If an electrically conducting continuum (solid, liquid, gas or plasma) be moving and placed before a magnetic field, the motion of the continuum is changed by the influence of the magnetic field and the magnetic field is also perturbed by the motion of the continuum: one affects the other and vice versa. This is interlocking in character. The motion of the conducting fluid across the magnetic field generates electric currents which modify the flow of the fluid. MHD effects in conducting liquids have been studied in the Laboratory by Hartmann and Williams.

They described how the viscosity of mercury seems to be enhanced when the flow takes place across a strong magnetic field. Also, the effects had been exploited in the case of molten sodium moving in a magnetic field and in the design of electromagnetic pumps and flow meters.

1-9 THE BASIC EQUATIONS OF MAGNETO HYDRODYNAMICS

Consider a fluid which has the property of electrical conduction, and suppose also that magnetic fields are prevalent. The electrical conductivity of the fluid and the prevalence of magnetic fields contribute to effects of two kinds: first, by the motion of the electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute to changes in the existing fields; and second, the fact that the fluid elements carrying currents transverse magnetic lines of force contributes to additional forces acting on the fluid elements. It is this twofold interaction between the motions and the fields that is responsible for patterns of behaviour which are often unexpected and striking.

We shall now write down the basic equations which express the interactions between the fluid motions and the magnetic fields. These are, of course, contained in Maxwell's equations and in the equations of hydrodynamics suitably modified. There is, however, one basic simplification which is possible. Since we have not been concerned with effects which are related in only way to the propagation of electromagnetic waves, we can ignore the displacement currents in Maxwell's equations. Closely related to this approximation is the further possibility of avoiding any explicit reference to the charge density. The reason for this is not that it is small in itself, but rather that its variations affect the equation expressing the conservation of charge only by terms of order $\frac{\mu^2}{c^2}$ and terms of this order we can legitimately ignored with the displacement currents ignored, Maxwell's equations are

$$\text{div } \overline{H} = 0 \qquad \text{........(9.01)}$$

$$\text{curl } \overline{H} = 4\pi\overline{J} \qquad \text{........(9.02)}$$

$$\text{curl } \overline{E} = -\mu\frac{\partial\overline{H}}{\partial t} \qquad \text{........(9.03)}$$

where in electromagnetic units, $\overline{E}$ and $\overline{H}$ are the intensities of the electric and magnetic fields, $\overline{J}$ is the current density, and μ is the magnetic permeability. The magnetic permeability will be taken as unity in all applications; it is retained only to identify the units. To complete the equations for the field, we need an equation for the current density. This requires some assumption concerning the nature of the fluid. In this thesis assumption will be made that the fluid may be considered as continuous and that the macroscopic properties need be taken into account only directly through the effects of viscosity and the heat and electrical conductivities. And the coefficients expressing these later effects will in tern be defined only phenomenologically.

Consider a fluid element, it has a velocity $\overline{q}$, the electric field it will experience is not $\overline{E}$, as measured by a stationary observer, but $\overline{E} + \mu\overline{q} \times \overline{H}$. If in accordance with our assumptions we suppose that a coefficient of electrical conductivity σ can be defined, then the current density will be given by

$$\overline{J} = \overline{\sigma}\left(E + \mu\overline{q}_{\wedge}\overline{H}\right) \qquad \text{........(9.04)}$$

Equations (9.01) – (6.04) are the basic equations of the field appropriate for hydromagnetics.

Through the occurrence of the velocity $\bar{q}$ in the expression for $\bar{J}$, the equations incorporate the effect of fluid motions on the electromagnetic field. The inverse effect of the field on the motions results from the force which the fluid elements experience in virtue of their carrying currents across magnetic lines of force. This is the Lorentz force given by

$$\mathcal{L} = \mu \bar{J} \times \bar{H} \qquad \text{........(9.05)}$$

or, according to equation (9.02)

$$\mathcal{L} = \frac{\mu}{4\pi} \operatorname{curl} \bar{H} \times \bar{H}. \qquad \text{........(9.06)}$$

In tensor notation L can be written as

$$\mathcal{L}_i = \frac{\mu}{4\pi} \in_{ijk} \in_{jlm} \frac{\partial H_m}{\partial x_\ell} H_k$$

$$= \frac{\mu}{4\pi} (\delta_{im}\delta_{kl} - \delta_{il}\delta_{km}) \frac{\partial H_m}{\partial x_\ell} \times H_k$$

$$= \frac{\mu}{4\pi} H_k \left(\frac{\partial H_i}{\partial x_k} - \frac{\partial H_k}{\partial x_i} \right) \qquad \text{........(9.07)}$$

Since H_i is solenoidal, we an also write

$$L_i = -\frac{\partial}{\partial x_i} \left(\mu \frac{|\bar{H}|^2}{8\pi} \right) + \frac{\partial}{\partial x_k} \left(\frac{\mu}{4\pi} H_i H_k \right). \qquad \text{........(9.08)}$$

]Last form for the Lorentz force expresses it as the sum of a hydrostatic pressure, $\frac{\mu|\overline{H}|^2}{8\pi}$, and a tension, $\frac{\mu|\overline{H}|^2}{4\pi}$, along the lines of force, or, equivalently as the sum of a pressure $\frac{\mu|\overline{H}|^2}{8\pi}$, transverse to the lines of force and a tension, $\frac{\mu|\overline{H}|^2}{8\pi}$, along the lines of force.

Including the Lorentz force among the other forces acting on the fluid, we have the equation of motion as

$$\rho\frac{\partial\overline{q}}{\partial t} = \operatorname{div}T + \rho\overline{F} + \mu\overline{J}\times\overline{H}, \qquad \text{........(9.09)}$$

where T is the total stress tensor and $\overline{F}$ includes the external forces of non-electromagnetic origin. The term $\nabla.T$ in (9.09) can be written as

$$(\nabla.T)_i = \frac{-\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial x_j}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu\frac{\partial u_j}{\partial x_j}\right).$$

For an incompressible fluid, the equation of motion takes the explicit form

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} - \frac{\mu H_j}{4\pi\rho}\frac{\partial H_i}{\partial x_j} = -\frac{\partial}{\partial x_i}\left(\frac{P}{\rho} + \mu\frac{|\overline{H}|^2}{8\pi\rho}\right) + \nu\nabla^2 u_i\,, \qquad \text{........(9.10)}$$

where the form (9.08) for the Lorentz force has been used.

We shall now obtain an equation of motion for the magnetic field.

According to equation (9.04)

$$\overline{E} = \frac{1}{\sigma}J - \mu\overline{q}_{\wedge}\overline{H} \qquad \text{........(9.11)}$$

or, making use of equation (9.02), we have

$$\overline{E} = \frac{1}{4\pi\sigma}\text{curl}\,\overline{H} - \mu\overline{q}_{\wedge}\overline{H}. \qquad \text{........ (9.12)}$$

Inserting this expression for $\overline{E}$ in (9.03), we obtain

$$\frac{\partial\overline{H}}{\partial t} - \text{curl}\left(\overline{q}_{\wedge}\overline{H}\right) = \text{curl}\left(\eta\,\text{curl}\,\overline{H}\right), \qquad \text{........(9.13)}$$

where $\eta = \dfrac{1}{4\pi\mu\sigma}$ ....(9.14)

we shall call η the resistivity and is of the dimension of $cm^2\,sec^{-1}$.

Equation (9.13) is entirely general; in particular, it is not restricted to incompressible fluids. If η is assumed to be a constant, equation (9.13) takes the form

$$\frac{\partial H_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_j H_i - u_i H_j\right) = \eta\nabla^2 H_i. \qquad \text{........(9.15)}$$

The elimination of $\overline{E}$ to obtain an equation for $\overline{H}$ represents an important simplification. Since the Lorentz force is also expressed in terms of $\overline{H}$ only, it follows that in the subsequent analysis we need not make, any further electric field.

A new general empirical model of visco-elastic fluid has been suggested by Sengupta and Kundu in the following form

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij}$$

$$\left(1+\sum_{j=1}^{n}\lambda_j\frac{\partial^j}{\partial t^j}\right)\tau'_{ij}=2\mu\left(1+\sum\mu_j\frac{\partial^j}{\partial t^j}\right)e_{ij}$$

$$e_{ij}=\frac{1}{2}\left(v_{i,j}+v_{j,i}\right),$$

where τ_{ij} is the stresstensor, τ'_{ij} is the deviatoric stresstensor, e_{ij} the rate of strain tensor, p the fluid pressure, λ_j are new material constants of which the greatest λ_1 represents the relaxation time parameter and $\lambda_2, \lambda_3..........\lambda_n$ are additional material constants; μ_j are also new material constants of which the greatest value μ_1 represents the strain rate retardation time parameter and $\mu_2, \mu_3........\mu_n$ are additional material constants representing the behaviour of a very wide class of visco-elastic liquids, δ_{ij} the metric tensor in Cartesin co-ordinates and μ, the coefficient of viscosity and v_i, the velocity components. The material constants λ_j and μ_j designating visco-elasticity satisfy the following conditions $\lambda_1 > \lambda_2 > \lambda_3 > .... > \lambda_n > 0$ and $\mu_1 > \mu_2 > \mu_3 > .... > \mu_n > 0$. i.e. they are arranged in descending order of magnitudes.

1-10 DISCUSSION ABOUT PAST RESEARCH RELEVANT TO THIS THESIS

Chao-Hosung [2] studied the stability of a rotating flow in the presence of an axial and a toroidal magnetic field. He investigated the stability with respect to non-axisymmetric perturbations of an inhomogeneous incompressible fluid rotating between two perfectly conducting, infinite, co-axial cylinders and established sufficient conditions for stability. We have studied this problem in addition to viscous part and establish sufficient conditions by separating real and imaginary parts. K. Ganguly and S. Gupta [12] studied the Hydromagnetic stability of helical flows. They investigated the stability of a steady non-dissipative MHD helical flow with velocity components $(0, r\Omega(r), W(r))$of an incompressible and inviscid fluid permitted by a helical magnetic field $(0, H_0(r), H_z(r))$ between two concentric cylinders. They established sufficient conditions. We have investigated this problem in case of viscous fluid and established sufficient conditions. Md. S. Islam and A.H. Beg [40] studied the invicid stratified parallel flows varying it two dimensions in presence of vertical magnetic field and established sufficient conditions. P.R. Sengupta, Bazlur Rahman and Dipak Kumar Kandar [41] studied unsteady viscous flow between two parallel flat plates. They considered the time varying pressure gradient and solved it by using Laplace transformation method. They showed the velocity profiles in graphically for different Reynolds numbers. We have studied the flow of a highly conducting (such as mercury) viscous incompressible fluid which is flowing between two parallel non-conducting planes in a uniform transverse magnetic field perpendicular to the planes. Expression for velocity profiles have been discussed with the help of tables and graphs for different values of Hartmann numbers. P.R. Sengupta, Bazlur Rahman and Dipak Kumar Kandar also in 2000 investigated the hydromagnetic flow of visco-elastic Rivlin-Ericksen fluid flowing down an inclined plane. In the same year P.R. Sengupta and Shyamal Kumar Kundu and Swarnakamal Misra [42] investigated the unsteady MHD flow of a visco-elastic Revlin-Ericksen fluid with transient pressure gradient through a rectangular channel. We have investigated same fluid through a uniform circular cylinder in a uniform transverse magnetic field. Here, the velocity profile of a fluid element of the problem has been calculated theoretically and graphically. P.R. Sengupta and Pijush Basak [43] studied the unsteady unidirection flow of an electrically conducting visco-elastic fluid of Oldroyd type between two parallel plates under the action of a transverse uniform magnetic field. They discussed the stability of the velocity profiles. We have investigated the unsteady MHD flow of visco-elastic Oldrlyd fluid with time varying body forced through rectangular channel. Here we have calculated the velocity profile of a fluid element of the problem theoretically and graphically and also discussed the stability of the velocity profile.

The equations of hydrodynamics allow some simple patterns of flow (Such as between parallel planes, concentric cylinders or rotating cylinder) as stationary solutions. The patterns of flow can be realized only for certain range's of the parameters characterizing them. Outside these ranges, they can not be realized. The reason for this lies in their inherent instability, that is in their inability to sustain themselves against small perturbations to which any physical system is subject. It is in the differentiations of the state from the unstable patterns of permissible flows stability originate. The class of such problems of stability has been enlarged by the interest in hydrodynamic flow of electrically conducting fluids in the presence of magnetic fields. This is the domain of hydromagnetics. Rayleigh [7], Goldstein [19], Lamb [20], Alfven [26], Milne-Thomson [21] and Batchelor [24] were the first who started investigating hydrodynamic stability flows. Also Carslaw and Jaegar [33], Michael [8], Cowling [27], Ferraro [28], Jeffery [29] and Cabannes [30] discussed hydromagnetic stability flows. Although vast research materials in hydrodynamic and hydromagnetic flows are now available, yet many problem in the hydrodynamic and hydromagnetic flows are to be analyzed.

This doctoral thesis is mainly devoted to a consideration of some typical problems in hydromagnetic stability. The mathematical treatment of problem in stability generally proceeds along the following lines.

The analysis in terms of normal modes and the analysis in terms of non-linear form.

We start from an initial flow which represent a stationary state of the system and suppose that the various physical variables describing the flow suffer small increments. In obtaining these equations from the relevant equations of motion we neglect all products and powers of the increment and retain only terms which are linear in them. The theory derived on the basis of such linearized equations is called the linear stability theory in construct to non-linear theories which attempt to allow for the finite amplitudes of perturbations. Stability means stability with respect to all possible disturbances. In practice, this is accomplished by expressing an arbitrary disturbance as a super position of certain basic possible modes and examining the stability of the system with respect to each of the modes. And we have mainly investigated those viscous flow problems which are analytically tractable.

In practical cases, such as arise in the design of ships, air craft, under water projectiles etc, it is usually necessary to carry out experiments on model and the full scale body should be geometrically similar, but also as far as possible, one should ensure that the two system posses dynamical similarity in the sense that the ratio's corresponding dynamical quantities at corresponding stations should be same.

Earlier some works were done in the case of inviscid fluid. One of the most important contribution in this research work is that the earlier investigations have been extended to the case of viscous fluid. If the system of hydromagnetic equations of motions have a time independent solution V(x,y,z), P(x,y,z), H(x,y,z) for the components of velocity, pressure and magnetic field, then this state is known as basic state. If an infinitesimal disturbance is superimposed on the basic state and if the solution approaches to the above steady state solution as time $t \to \infty$, we say the system is stable, otherwise it is unstable. To study the hydromagnetic stability of some flows due to small disturbance, the resultant linear system of equations contain time t only through derivatives with respect to t. This then leads us to conclude that any solution in general may be expected to contain exponential time factor $e^{-\omega t}$. If all the characteristic values of ω have positive real parts only, the motion is stable with respect to infinitesimal disturbance, but even if only one characteristic value has negative real part then the motion becomes unstable.

The hydromagnetic instabilities of an inviscid flow between two concentric cylinders which has only swirl velocity component $u_\theta(r)$ in the direction of increasing azimuthal angle θ for axisymmetric disturbance has been well understood. Since Rayleigh [7] gave this criterion that a necessary and sufficient condition for stability that the square circulation (rV) should no where decrease as r increases from inner to the outer cylinder. Rayleigh's remark has a strong analogy with the stability of a density stratified fluid at rest under the action of gravity so long as only axisymmetric perturbation is concerned. Chandrasekhar [5] has considered the same problem by considering both axial and swirl component alone. But if the fluid is taken as perfect electric conductor and subject to a transverse magnetic field, then in the case of zero axial flow that magnetic field has an effect similar to that of swirl velocity and that of Rayleigh's criterion.

Howard and Gupta [4] extended this problem by investigating the hydromagnetic, instability with respect to axisymmetric disturbances of a steady non-dissipative helical flows with velocity components $(0, u_\theta(r), u_z(r))$ in the r, θ, z directions respectively of a conducting fluid permeated by an axial or azimuthal magnetic field. However enough investigations have not been done when the disturbance is non-axismmetry case the mathematical complexities arise due to non-axisymmetric disturbance in viscous fluid. In the absence of magnetic diffusion non-axisymmetric disturbances generally twist the magnetic lines of force and produce an intimate coupling between the hydrodynamic and hydromagnetic effects. Axisymmetric disturbance on the other hand, only bend the lines of force and under certain circumstance hydrodynamic effects may dominate over those due to magnetic forces. Exploiting this idea, Ganguly and Guta [12] investigated the instabilities of non-dissipative helical flow of an incompressible conducting fluid permeated by helical field $(0, B_0(r), B_z(r))$ for non-axisymmetric disturbances using a technique due to Barston [6]. Acheson [11] examined a class of hydromagnetic instabilities in a uniform rotating homogeneous incompressible fluid that arises due to the variations of the azimuthal magnetic intensity with distances from the axis of rotation. In a subsequent paper Acheson examined the instability of a radially stratified fluid rotating between two co-axial cylinders with particular emphasis on the case when the angular velocity greatly exceeds both buoy and Alfven frequencies.

In non-axisymmetric perturbation for instability and wave like character the magnetic field should be pre-dominantly azimuthal, various bounds on the phase speeds and growth rates were derived. We noted that there was a strong tendency to propagate against the basic rotation. During the same period C.H. Sung [2] derived sufficient condition for stability for a rotating inviscid fluid when $B_0 \neq 0$ and $B_z = 0$ and in particular case like small rotation, rapid rotation and strong magnetic field when $B_0 \neq 0 \neq B_z$.

To the best of author's knowledge the above problems have not been solved for the rotation of a viscous fluid. The author has successfully investigated these unsolved problem and produced hydromagnetic instability condition.

We have however used the transformation and the limitations earlier adopted by Sung [2] who used inner product method for axisymmetric disturbances. He deduced the characteristic equation in the form

$$\sigma^2 \underline{\xi} - 2i\sigma F \underline{\xi} - Q \underline{\xi} = 0 .$$

He had shown the inner products $<\underline{\xi}, iF\underline{\xi}>$ and $<\underline{\xi}, Q\underline{\xi}>$ were both real due to the Hermitian property of the operators iF and Q. This led him to deduce the sufficient stability condition everywhere which was similar to the condition first derived by Howard and Gupta [4] for ρ and B_z constant and in the absence of acceleration due to gravity. But in the present analysis for axisymmetric disturbances of viscous fluid, the corresponding operators involved are no longer Hermitian and hence the inner products are not real. Accordingly the stability analysis of viscous fluid becomes a more complex problem. However, we have overcome this by separating real and imaginary parts of the inner products.

There are circumstances to consider a large variety of continua in which considerable impetus is given to the development of study of material properties exhibiting both the properties of ideal elastic bodies and those of viscous liquids. It constitutes the subjects of the theory of elasticity and hydromechanics of viscous liquids. In fact, there are materials, solid or liquid, which exhibit the properties of elasticity of solids and viscosity of liquids. It gives rise to the discipline of Rheology of continua, the continua may be solid, liquid or gases. These liquids are some times called as non-Newtonian liquids or non-Newtonian fluids or visco-elastic fluids. In this analysis we have also investigated the stability of a visco-elastic Oldroyd fluid between two concentric cylinders in presence of a uniform transverse magnetic field. The fluid is assumed to be incompressible and dissipative. But Hurwitz stability criteria for a small disturbing forces is that of the deviation is small from the initial condition of motion. In case of application of transient body force $X = X_0 e^{-\omega t}$ the motion of the fluid will be stable if $\omega > 0$. It means that the motion of the fluid tends to finite value or zero as t tends to infinity and hence is stable. Both the roots of the characteristic equation in ω are to be positive for motion to be stable. The presence of a negative root would lead to an unstable transient motion. Also if the roots are complex conjugate with positive real part, the motion will be stable and damped oscillation, while the complex conjugate roots with negative real part will generate instability.

In our problem we have the relation

$$K^2 = \frac{(1-\lambda\omega)(\omega - M^2)}{1-\lambda_2\omega}.$$

This leads to the characteristic equation as

$$\lambda_1\omega^2 - \left(1+\lambda_1 M^2 + \lambda_2 K^2\right)\omega + K^2 + M^2 = 0.$$

The roots are $\omega = \dfrac{S \pm \sqrt{S^2 - 4\lambda_1(K^2+M^2)}}{2\lambda_1}$,

where $S. = 1 + \lambda_1 M^2 + \lambda_2 K^2$.

The transient motion will be stable if

$$S^2 - 4\lambda_1(K^2+M^2) > 0 \text{ and}$$

$$S - \sqrt{S^2 - 4\lambda_1(K^2+M^2} > 0.$$

These two relations implies

$$\lambda_1 > \lambda_2 > 0,$$

$$0 < \omega < \frac{1}{\lambda_1} + M^2\left(1 - \frac{\lambda_2}{\lambda_1}\right)$$

and $M^2 < \dfrac{1}{\lambda_2}$.

These are the conditions of stable of visco-elastic Oldroyd type fluid.

CHAPTER - II

HYDROMAGNETIC STABILITIES OF A ROTATING VISCOUS INCOMPRESSIBLE FLUIDS

2-1 INTRODUCTION

We have investigated a magneto hydrodynamics stability with respect to axisymmetric perturbations of an inhomogeneous incompressible viscous fluid rotating between two perfectly conducting, infinite co-axial cylinders in presence of an axial and a toroidal magnetic field. Acheson [1] studied the stability of a uniform rotating cylindrical flow in the presence of a magnetic field. Sung [2] investigated the stability of a rotating flow of an inhomogeneous incompressible fluid in presence of an axial and a toroidal magnetic filed with respect to non axisymmetric perturbation between two co-axial infinite cylinders and he neglected the viscosity of the fluid and used inner product method for axisymmetric perturbations and without normal mode assumption Barston [6] deduced that the sufficient stability conditions.

For axisymetric perturbations according to Sung the characteristics equation was

$$\sigma^2\underline{\xi} - 2i\sigma p\underline{\xi} - Q\underline{\xi} = 0$$

where,

$$p\underline{\xi} = -\Omega(e_r\xi_\theta - e_\theta e_r)$$

and

$$Q\underline{\xi} = \rho^{-1}\nabla(\delta\pi) + e_r[N^2 + (\varphi - 4\Omega^2) - rH_r]\xi_r$$

He shew the inner product $(\underline{\xi}, ip\underline{\xi})$ and $(\underline{\xi}, Q\underline{\xi})$ were both real due to the Hermitian property of the operators ip and Q. This led him to deduce the sufficient condition, first derived by Howard and Gupta [4] for ρ and B_z constant and in the absence of acceleration due to gravity. This is known as Howard-Gupta criterion for stability.

In this paper, we have investigated magneto hydrodynamics stability with respect to axisymmetric perturbation of an inhomogeneous incompressible, viscous fluid rotating between two perfectly conducting infinite co-axial cylinders in presence of an axial and a toroidal magnetic field. But in the present analysis with respect to axisymmetric perturbations of viscous fluid, the corresponding operators involved are not Hermitian and so as the inner-product are no longer real. Accordingly, the stability analysis of a rotating viscous fluid becomes a more complex problem. However, we have overcome this by separating the real and imaginary parts of the inner products.

2-2 MATHEMATICAL FORMULATION

Let us consider a viscous fluid rotating between two perfectly conducting infinite coaxial cylinders in the presence of an axial and a toroidal magnetic field. The fluid assumes to be incompressible but inhomogeneous and the dissipative mechanisms such as magnetic resisitivity and thermal diffusivity are neglected. The governing equations of motion in cylindrical co-ordinates (r,θ,z) are

$$\frac{\partial \bar{q}}{\partial t}+(\bar{q}.\nabla)\bar{q}=\frac{-1}{\rho}\nabla\pi+\frac{1}{\rho}(\bar{B}.\nabla)\bar{B}-g+\nu\nabla^{2}\bar{q} \qquad \text{.........(2.01)}$$

$$\frac{\partial \bar{B}}{\partial t}=\nabla_{\wedge}(\bar{q}_{\wedge}\bar{B}) \qquad \text{.........(2.02)}$$

$$\nabla.\bar{q}=0 \qquad \text{.........(2.03)}$$

$$\nabla.\bar{B}=0 \qquad \text{.........(2.04)}$$

$$\frac{\partial \rho}{\partial t}+(\bar{q}.\nabla)\rho=0 \qquad \text{.........(2.05)}$$

where, $\bar{q}$ is the velocity of the fluid, ρ the density, $\bar{g}$ is the gravitational acceleration, $\pi=P+\frac{1}{2}(\bar{B}.\bar{B})$ is the total pressure, $\bar{B}$ is the magnetic field and ν the kinetic co-efficient of viscosity.

The equilibrium state is derived by

$$q_0 = (0, r\Omega, W(r)),$$

$$B_0 = (0, B_\theta(r), B_z(r)),$$

$$\rho_0 = \rho_0(r)$$

and

$$g_0 = (g_r, 0, 0).$$

which gives

$$g_r - r\Omega^2 = G_r - \frac{1}{2} r^2 H_r - 2rv_\theta^2 - \rho_0^{-1} B_z D B_z, \qquad \ldots\ldots(2.06)$$

where

$$D = \frac{d}{dr},$$

$$G_r = -\rho^{-1} D\rho,\ V_\theta^2 = \frac{B_\theta^2}{\rho r^2},\ V_z^2 = \frac{B_z^2}{\rho r^2},$$

$$H_r = \frac{1}{\rho} D\left(\frac{B_\theta}{r}\right)^2 = DV_\theta^2 + \frac{1}{\rho}(D\rho)V_\theta^2 \text{ and}$$

$$rD^2\Omega + 3D\Omega = 0 \qquad \ldots\ldots(2.07)$$

$$rD^2 W + DW = 0. \qquad \ldots\ldots(2.08)$$

(2.07) and (2.08) implies

$$\Omega = A + \frac{B}{r^2} \text{ and } W = C + D\log r.$$

Let us consider an infinitesimal perturbation of the viscous flow represented by the above stationary state and let the perturbation state be represented by

$$q_0 + \delta q = (U_r, r\Omega + U_\theta, W + U_z), \pi + \delta\pi,$$

$$B_0 + \delta B = (b_r, B_\theta + b_\theta, B_z + b_z), \rho_0 + \delta\rho$$

The perturbation quantities are assumed very small. Then the linearized equations of motion are

$$\frac{\partial U_r}{\partial t} + \Omega\frac{\partial U_r}{\partial \theta} + W\frac{\partial U_r}{\partial z} - 2\Omega U_\theta + \frac{1}{\rho_0}(g_r - r\Omega^2)\rho$$
$$= \frac{-1}{\rho_0}\frac{\partial}{\partial r}(\delta\pi) + \frac{1}{\rho_0}\left(\frac{B_\theta}{r}\frac{\partial b_r}{\partial \theta} + B_z\frac{\partial b_r}{\partial z} - 2\frac{B_\theta b_\theta}{r}\right) + \nu\left(\nabla^2 U_r - \frac{2}{r^2}\frac{\partial U_\theta}{\partial \theta} - \frac{U_r}{r^2}\right) \quad \text{....(2.09)}$$

$$\frac{\partial U_\theta}{\partial t} + \Omega\frac{\partial U_\theta}{\partial \theta} + W\frac{\partial U_\theta}{\partial z} + 2\Omega U_r + rU_r D\Omega = \frac{-1}{\rho_0 r}\frac{\partial}{\partial \theta}(\delta\pi)$$
$$+ \frac{1}{\rho_0}\left(b_r DB_\theta + \frac{B_\theta}{r}\frac{\partial b_\theta}{\partial \theta} + B_z\frac{\partial b_\theta}{\partial z} + \frac{B_\theta b_r}{r}\right) + \nu\left(\nabla^2 U_\theta + \frac{2}{r^2}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta}{r^2}\right) \quad \text{......(2.10)}$$

$$\frac{\partial U_z}{\partial t} + \Omega\frac{\partial U_z}{\partial \theta} + W\frac{\partial U_z}{\partial z} = \frac{-1}{\rho_0}\frac{\partial}{\partial z}(\partial\pi) + \frac{1}{\rho_0}\left(b_r DB_z + \frac{B_\theta}{r}\frac{\partial b_z}{\partial \theta} + B_z\frac{\partial b_z}{\partial z}\right) + \nu\nabla^2 U_z \quad \text{...(2.11)}$$

In the linearized equations of motion we shall assume that g_r depends on the radial distance r, its Eulerian variation may be neglected $\partial g = 0$.

Assume a normal mode solution for the Lagrangian displacement

$$\xi = \xi(r,\theta,z)e^{i\sigma t} = \xi(r)e^{i(m\theta + kz + \sigma t)} \quad \text{........(2.12)}$$

and noting that the Larangian operator Δ and Eulerion operator ∂ are related by

$$\Delta = \delta + \bar{\xi}.\nabla \text{ that is } \Delta\bar{q} = \delta\bar{q} + (\bar{\xi}.\nabla)\bar{q}.$$

And considering

$$\left.\begin{aligned} U_r &= i\omega\xi_r \\ U_\theta &= i\omega\xi_\theta - r\xi_r D\Omega \\ U_z &= i\omega\xi_z - \xi_r DW \end{aligned}\right] \qquad \text{........(2.13)}$$

where

$$\omega = \sigma + m\Omega + KW .$$

The linearized equation (2.02) becomes

$$\left.\begin{aligned} b_r &= \frac{imB_\theta}{r}(1+\beta)\xi_r \\ b_\theta &= \frac{imB_\theta}{r}(1+\beta)\xi_\theta + \left(\frac{B_\theta}{r} - DB_\theta\right)\xi_r \\ b_z &= \frac{imB_\theta}{r}(1+\beta)\xi_z - (DB_z)\xi_r \end{aligned}\right] \qquad \text{........(2.14)}$$

where

$$\beta = \frac{rKB_z}{mB_\theta} .$$

And the linearized form of equation (2.05) is given by

$$i(\sigma + m\Omega + KW)\rho_0 + u_r\rho_0{}' = 0 ,$$

$$\xi_r = -\frac{\rho_0}{\rho_0{}'} \qquad \text{........(2.15)}$$

With the help of normal mode solution (2.13) and equations (2.14), equations (2.09), (2.10) and (2.11) respectively becomes

$$-\omega^2\xi_r - 2i\omega\Omega\xi_\theta + \{N^2 + (\varphi - 4\Omega^2) - rH_r\}\xi_r + \frac{1}{\rho}\frac{\partial}{\partial r}(\partial\pi) + m^2 V_\theta^2(1+\beta)\xi_r$$

$$+ 2imV_\theta^2(1+\beta)\xi_\theta - \nu L(i\omega\xi_r) + \frac{2im\gamma(i\omega\xi_\theta)}{r^2} - \frac{2im\nu D\Omega}{r}\xi_r = 0. \qquad \text{........(2.16)}$$

$$-\omega^2\xi_\theta - 2i\omega\Omega\xi_r + \frac{1}{\rho r}\frac{\partial}{\partial\theta}(\delta\pi) - 2im(1+\beta)V_\theta^2\xi_r + m^2(1+\beta)^2 V_\theta^2\xi_\theta - \nu L(i\omega\xi_\theta - r\xi_r D\Omega)$$

$$-\frac{2im\nu}{r^2}(i\omega\xi_r) = 0. \qquad \text{........(2.17)}$$

$$-\omega^2\xi_z + \frac{1}{\rho}\frac{\partial}{\partial z}(\partial\pi) + m^2(1+\beta)^2 V_\theta^2\xi_z - \nu L(i\omega\xi_z - \xi_r DW) - \frac{\nu}{r^2}(i\omega\xi_z - \xi_r DW) = 0.$$

........(2.18)

The subscript zero used to indicate stationary state in (2.06) has been dropped in (2.16) and will be dropped here after,

where

$$M^2 = \left(\frac{m^2}{r^2}\right) + K^2,\ L = DD^* - M^2.$$

Rayleigh [7] discriminant $\varphi = r^{-3}D(r^4\Omega^2)$ and Brunt Vaisala frequency N is given by

$$N^2 = -\rho^{-1}D\rho(g_r - r\Omega^2).$$

Multiplying (2.16), (2.17), (2.18) respectively by e_r, e_θ, e_z and adding we get

$$\omega^2\underline{\xi} - 2\omega iB\underline{\xi} - C\underline{\xi} = 0, \qquad \text{........(2.19)}$$

where

$$iB\underline{\xi} = -i\left[\left(\Omega - \frac{imv}{r^2}\right)(\xi_\theta e_r - \xi_r e_\theta) + \frac{v}{2}L\underline{\xi} + \frac{v}{2r^2}\xi_z e_z\right] \qquad \text{........(2.20)}$$

and

$$C\underline{\xi} = \rho^{-1}\nabla(\delta\pi) + m^2 V_\theta^2(1+\beta)^2\underline{\xi} + 2imV_\theta^2(1+\beta)(\xi_\theta e_r - \xi_r e_\theta). \qquad \text{........(2.21)}$$

From (1.14) and using

$$\partial\bar{q} = i\omega\underline{\xi} - e_\theta(rD\Omega)\xi_r - e_z(DW)\xi_r,$$

We get

$$\delta B = \frac{imB_\theta}{r}(1+\beta)\underline{\xi} - e_\theta\left(DB_\theta - \frac{B_\theta}{r}\right)\xi_r - e_z(DB_z)\xi_r. \qquad \text{........(2.22)}$$

The linearized version of the boundary conditions that radial components U_r of velocity vanish at both walls i.e. $\xi_r = 0$, at $r = r_1, r_2$

The linearized continuity equation $\nabla.\bar{q} = 0$ implies

$$\nabla.\underline{\xi} = D\xi_r + \frac{1}{r}\xi_r + \frac{im}{r}\xi_\theta + iK\xi_z = 0 \qquad \text{........(2.23)}$$

Equations (2.19) – (2.22) constitute the basic equation of the stability analysis.

Since the frequency ω in the rotating frame is constant, the equation of the form $\omega^2\underline{\xi} - 2i\omega B\underline{\xi} - C\underline{\xi} = 0$ is convenient for the case of uniform rotation. For differential rotation it is more convenient to rewrite (2.19) as

$$\sigma^2\underline{\xi} - 2i\sigma F\underline{\xi} - G\underline{\xi} = 0, \qquad \text{........(2.24)}$$

where

$$iF\underline{\xi} = iB\underline{\xi} - (m\Omega + KW)\underline{\xi} \qquad \text{........(2.25)}$$

and

$$G\underline{\xi} = C\underline{\xi} + 2(m\Omega + KW)iB\underline{\xi} - (m\Omega + KW)^2\underline{\xi} \qquad \text{........(2.26)}$$

2-3 STABILITY OF AXISYMMETRIC PERTURBATION (m=0)

In this case $iF\underline{\xi}$ and $G\underline{\xi}$ in the characteristic equation becomes

$$iF\underline{\xi} = -i\left[\Omega(\xi_\theta e_r - \xi_r e_\theta) + \frac{\nu}{2}L\underline{\xi} + \frac{\nu}{2r^2}\xi_z e_z\right] - kW\xi \qquad \text{........(3.01)}$$

and

$$G\underline{\xi} = \rho^{-1}\nabla(\partial\pi) + \frac{K^2B_z^2}{\rho}\underline{\xi} + \frac{2iKB_\theta B_z}{\rho r}(\xi_\theta e_r - \xi_r e_\theta) + e_r\left[N^2 + \left(\varphi - 4\Omega^2\right) - rH_r\right]\xi_r$$

$$-2i\nu KDW\underline{\xi} + \nu L\left(r\xi_r D\Omega\right)e_\theta + \nu L\left(\xi_r DW\right)e_z + \frac{e_z \nu \xi_r DW}{r^2} + 2iKWD\underline{\xi} - K^2W^2\underline{\xi}.$$

........(3.02)

2-4 INNER PRODUCT

Here iF and G are not Hermitian. This property is crucial to mention that stability always is not straight forward like the one carried out by using in the case of inviscid fluid. According to Sung [2] we define a inner product for any operation B as

$$\left(\underline{\xi}, B\underline{\xi}\right) = \int_R \rho\bar{\underline{\xi}}.B\underline{\xi}dx = \int_R \rho\bar{\underline{\xi}}.B\underline{\xi}, \qquad \text{........(4.01)}$$

where R is the region occupied by the fluid in the stationary state, $\bar{\underline{\xi}}$ is the complex conjugate of $\underline{\xi}$ and the last identity is introduced as short hand.

In our problem the stationary state is cylindrical (r, θ, z) that the fluid contain between two cylinders. Accordingly R is taken as the volume between two co-axial cylinders $r = r_1$ and $r = r_2$ by a proper length in z direction.

We can therefore write

$$(\underline{\xi}, B\underline{\xi}) = \int_{r_1}^{r_2} \rho \bar{\underline{\xi}}.B\underline{\xi}\, r dr \qquad \text{........(4.02)}$$

with the help of above definition of an inner product the growth rate of a small perturbations follows from the roots of the equation.

$$\sigma^2(\bar{\underline{\xi}}, \underline{\xi}) - 2\sigma(\bar{\underline{\xi}}, iF\underline{\xi}) - (\bar{\underline{\xi}}, G\underline{\xi}) = 0. \qquad \text{........(4.03)}$$

Now

$$I_1 = (\bar{\underline{\xi}}, \underline{\xi}) = \int \rho \bar{\underline{\xi}}.\underline{\xi},$$

which is real.

Again

$$(\bar{\underline{\xi}}, iF\underline{\xi}) = -i\int \rho\left[\Omega(\xi_\theta \bar{\xi}_r - \xi_r \bar{\xi}_\theta) + \frac{\nu}{2} L\bar{\underline{\xi}}.\underline{\xi} + \frac{\nu}{2r^2}\bar{\underline{\xi}}_z.\underline{\xi}_z\right] - \int \rho W \bar{\underline{\xi}}.\underline{\xi} = I_2 + iI_3, \qquad \text{........(4.04)}$$

where,

$$I_2 = \int \rho\{\Omega R_1 - W\bar{\underline{\xi}}.\underline{\xi}\} \qquad \text{........(4.05)}$$

$$I_3 = -\int \rho \frac{\nu}{2}\left[\bar{\underline{\xi}}.\underline{\xi} + \frac{1}{r^2}\bar{\xi}_z.\xi_z\right] \qquad \text{........ (4.06)}$$

and $\bar{\xi}_r \xi_\theta - \xi_r \bar{\xi}_\theta = iR_1$, R_1 is real.

Again

$$\left(\underline{\xi}, G\underline{\xi}\right) = K^2\int B_z^2|\underline{\xi}|^2 - 2K\int \frac{B_\theta B_z}{r} R_1 + \int \rho\left[N^2 + \left(\phi - 4\Omega^2\right) - rH_r\right]\left|\xi_r^2\right| - 2i\nu K\int \rho DW|\xi|^2$$

$$+ \nu\int \rho L\left(r\xi_r D\Omega\right)\bar{\xi}_\theta + \nu\int\left(\xi_r.DW\right)\bar{\xi}_z + \nu\int \frac{\rho\xi_r\bar{\xi}_z DW}{r^2} = I_4 + iI_5. \quad \ldots\ldots(4.07)$$

where,

$$I_4 = K^2\int B_z^2|\underline{\xi}|^2 - 2K\int \frac{B_\theta B_z}{r} R_1 + \int \rho\left[N^2 + \left(\phi - 4\Omega^2\right) - rH_r\right]|\xi_r|^2$$

$$+ \text{Real}\ \nu\int \frac{\rho\xi_r\xi_z DW}{r^2} + \text{Real}\ \nu\int \rho L\left(r\xi_r D\Omega\right)\bar{\xi}_\theta + \text{Real}\ \nu\int\left(\xi_r DW\right)\bar{\underline{\xi}}_z.$$

$$\ldots\ldots(4.08)$$

Thus we find that (4.03) is a quadratic equation in σ with complex co-efficient. It's roots are

$$\sigma = \frac{\left(\bar{\underline{\xi}}, iF\underline{\xi}\right) \pm \left[\left(\bar{\underline{\xi}}, iF\underline{\xi}\right)^2 + \left(\bar{\underline{\xi}}, \underline{\xi}\right)\left(\bar{\underline{\xi}}, G\underline{\xi}\right)\right]^{\frac{1}{2}}}{\left(\bar{\underline{\xi}}, \underline{\xi}\right)}$$

$$= \frac{I_2 + iI_3 \pm \left[I_6 + iI_7\right]^{\frac{1}{2}}}{I_1}$$

where

$$I_6 = I_2^2 - I_3^2 + I_1 I_4$$

and

$$I_7 = 2I_2 I_3 + I_1 I_5$$

Separating the real and imaginary parts of σ we have

$$\sigma_r = \frac{I_2 \pm \lambda^{\frac{1}{2}} \cos\frac{1}{2}\tan^{-1} y}{I_1}$$

and

$$\sigma_i = \frac{I_3 \pm \lambda^{\frac{1}{2}} \sin \frac{1}{2} \tan^{-1} y}{I_1}$$

where

$$\lambda^2 = I_6^2 + I_7^2 \text{ and } y = \frac{I_7}{I_6}.$$

2-5 CONCLUSION

For the propagation of unstable mode $\sigma_i \neq 0$

$$\sigma_i = \frac{I_3 \pm \left\{ \lambda^{\frac{1}{2}} \left(-1 \pm \sqrt{1+y^2} \right) \right\} / \mu}{I_1}$$

where

$$\mu = \sqrt{y^2 + \left(1 + \sqrt{1+y^2}\right)^2}$$

The following are the possible values of σ_i

$$\sigma_{i_1} = \frac{1}{I_1} \left[I_3 + \left\{ \lambda^{\frac{1}{2}} \left(-1 + \sqrt{1+y^2} \right) \right\} / \mu \right]$$

$$\sigma_{i_2} = \frac{1}{I_1} \left[I_3 + \left\{ \lambda^{\frac{1}{2}} \left(-1 - \sqrt{1+y^2} \right) \right\} / \mu \right]$$

$$\sigma_{i_3} = \frac{1}{I_1} \left[I_3 - \left\{ \lambda^{\frac{1}{2}} \left(-1 + \sqrt{1+y^2} \right) \right\} / \mu \right]$$

$$\sigma_{i_4} = \frac{1}{I_1}\left[I_3 - \left\{ \lambda^{\frac{1}{2}}\left(-1 - \sqrt{1+y^2}\right) \right\} / \mu \right]$$

From (4.06) it is clear that $I_3 < 0$

1. Propagation will be unstable, oscillatory or stable according as

$$I_3 \underset{<}{\overset{>}{=}} \left[\lambda^{\frac{1}{2}} \frac{\left(-1 + \sqrt{1+y^2}\right)}{\mu} \right]$$

2. σ_{i_2} and σ_{i_3} are always < 0, accordingly unstable mode will be propagated in this case.

3. Again propagation will be unstable, oscillatory or stable according as

$$I_3 \underset{<}{\overset{>}{=}} \left[\lambda^{\frac{1}{2}} \frac{\left(1 + \sqrt{1+y^2}\right)}{\mu} \right]$$

If the fluid is non viscous ($v = 0$) the characteristic equation (2.24) reduced to

$$\sigma^2 \underline{\xi} - 2i\sigma F \underline{\xi} - Q\underline{\xi} = 0,$$

which is obtained earlier by Sung [2]

CHAPTER - III

HYDROMAGNETIC STABILITY OF HELICAL FLOWS IN VISCOUS FLUID

3-1 INTRODUCTIN

We have investigated a MHD stability with respect to axisymmetric perturbations of an incompressible viscous fluid rotating between two perfectly conducting infinite co-axial cylinders in presence of an axial and transverse magnetic field. Rayleigh [7] studied the non dissipative flow of an incompressible fluid with circular streamlines between two concentric circular cylinders is stable with respect to axisymmetric disturbances if the square of circulation decreases no where in the radially outward direction. He found that this problem has a remarkable analogy with that of the stability of a density stratified fluid at rest under gravity. Michael [8] extended this problem to the case of a perfectly conducting fluid with an electric current disturbance parallel to the axis of the cylinders. He observed that in the presence of axisymmetric disturbances, Rayleigh's analogy still holds and the magnetic field due to current has an effect similar to that of the basic velocity. Howard and Gupta [4] studied the stability of steady non dissipative helical flow of a conducting fluid with an axial volume current. They observed that such a flow consisting of an azimuthal and an axial component of velocity would be stable with respect to axisymmetric disturbances if a suitable Richardson number based on the azimuthal component of the velocity, the circular magnetic field due to the current distribution and the shear in the axial flow exceeds $\frac{1}{4}$ everywhere in the flow. Agrawal [9] derived a sufficient condition for stability of the steady non dissipative helical flow of a conducting fluid permeated by an axial volume current and a uniform magnetic field for axisymmetric disturbance. In case of hydrodynamics, the study of stability of a helical flow subject to non axisymmetric disturbances derived by Pedley [10], he observed that flow in a rigidly rotating pipe becomes unstable with respect to helical perturbations in the limit of very rapid rotation. Chandrasekhar [5] studied MHD flows and derived very few general stability criteria for non-dissipative MHD flows. Acheson [11] showed that undisturbed motion of rigid rotation between two concentric cylinders permeated by either an azimuthal or an axial magnetic field, a non axisymmetric unstable mode propagated against the basic rotation. Acheson [1] also derived a quadrant theorem to localize the complex wave spead for an unstable mode of slow amplifying waves, the undisturbed state is one of the pure rigid rotation in the presence of an azimuthal magnetic field. K. Ganguly and A.S. Gupta [12], incase of helical magnetic field derived that the phase speed for any disturbance lies between to bounds which depends on the basic velocity distribution but not on the basic magnetic field. In absence of basic motion, no unstable mode can propagate in the system. According to Ganguly and Gupta [12], the characteristics equation $(\omega^2 p - \omega iA - H)\underline{\xi} + F_\xi = 0$ and the operators p, iA, and H are Hermitian. They shew the inner product $(\underline{\xi}, iA\underline{\xi})$ and $(\underline{\xi}, H\underline{\xi})$ were both real due to the Hermitian property of the operators. In this paper we have investigated magneto hydrodynamics stability with respect to axisymmetric perturbation of an incompressible viscous fluid rotating between two perfectly conducting infinite co-axial cylinders in presence of an axial and a transverse magnetic field. But in the present analysis with respect to axisymmetric perturbations of viscous fluid, the corresponding operators involved are not Hermitian and so as the inner product are no longer real. Accordingly, the extension of the inner product in the case of perturbations of viscous fluid is a more complex problem. We have derived sufficient conditions by separating the real and imaginary parts of the inner product.

3-2 MATHEMATICAL FORMULATION

Let us consider a viscous fluid rotating between two perfectly conducting co-axial cylinder in the present of an axial and transverse magnetic field. The fluid assumes to be incompressible and non-dissipative. The governing equations of motion in cylindrical co-ordinates (r, θ, z) are,

$$\frac{\partial u}{\partial t} + (\bar{q}.\nabla)u - \frac{v^2}{r} - \frac{1}{\rho}(\bar{B}.\nabla B_r - \frac{B_\theta^2}{r})$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu(\nabla^2 u - \frac{2}{r^2}\frac{\partial v}{\partial \theta} - \frac{u}{r^2}). \qquad \text{........(2.01)}$$

$$\frac{\partial v}{\partial t}+(\bar{q}.\nabla)v-\frac{uv}{r}-\frac{1}{\rho}(\bar{B}.\nabla B_\theta-\frac{B_r B_\theta}{r})$$

$$=-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}+\nu(\nabla^2 v+\frac{2}{r^2}\frac{\partial u}{\partial \theta}-\frac{v}{r^2}). \qquad \text{........(2.02)}$$

$$\frac{\partial w}{\partial t}+(\bar{q}.\nabla)w-\frac{1}{\rho}(\bar{B}.\nabla B_z)=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\nabla^2 W. \qquad \text{........(2.03)}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0. \qquad \text{........(2.04)}$$

$$\frac{\partial B_r}{\partial r}+\frac{B_r}{r}+\frac{1}{r}\frac{\partial B_\theta}{\partial \theta}+\frac{\partial B_z}{\partial z}=0. \qquad \text{........(2.5)}$$

$$\frac{\partial \bar{B}}{\partial t}=\nabla_\Lambda(\bar{q}\Lambda\bar{B}) \qquad \text{........(2.6)}$$

where $\bar{q}=(u,v,w)$ is the velocity of the fluid, p is the total pressure, $\bar{B}$ the magnetic field and ν the coefficient of viscosity.

The equilibrium state is derived by

$$q_0(0,r\Omega,0),B_0\{0,B_\theta(r),B_z(r)\}.$$

Let us consider an infinitesimal perturbation of the viscous flow represented by the above stationary state and let the perturbation state be representation by

$$q_0+\partial q=(u_r,r\Omega+u_\theta,u_z),$$

$$B_0+\partial B=(b_r,B_\theta+b_\theta,B_z+b_z)$$

and

$$p+\partial p.$$

The perturbation quantities are small. So the linearized equations of motions are

$$\frac{\partial u_r}{\partial t}+\Omega\frac{\partial u_r}{\partial \theta}-2\Omega u_\theta-\frac{1}{\rho}\left(\frac{B_\theta}{r}\frac{\partial b_r}{\partial \theta}+B_z\frac{\partial b_r}{\partial z}-\frac{2B_\theta b_\theta}{r}\right)$$

$$=-\frac{1}{\rho}\frac{\partial(\partial p)}{\partial r}+v\left(\nabla^2 u_r-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}-\frac{u_r}{r^2}\right). \qquad \text{........(2.07)}$$

$$\frac{\partial u_\theta}{\partial t}+\Omega\frac{\partial u_\theta}{\partial \theta}+(2\Omega+rD\Omega)u_r-\frac{1}{\rho}\left(\frac{B_\theta}{r}\frac{\partial b_\theta}{\partial \theta}+B_z\frac{\partial b_\theta}{\partial z}+b_r DB_\theta+\frac{B_\theta b_r}{r}\right)$$

$$=-\frac{1}{\rho r}\frac{\partial(\partial p)}{\partial \theta}+v\left(\nabla^2 u_\theta+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}\right). \qquad \text{........(2.08)}$$

$$\frac{\partial u_z}{\partial t}+\Omega\frac{\partial u_z}{\partial \theta}-\frac{1}{\rho}\left(\frac{B_\theta}{r}\frac{\partial b_z}{\partial \theta}+B_z\frac{\partial b_z}{\partial z}+b_r DB_z\right)=-\frac{1}{\rho}\frac{\partial(\partial p)}{\partial z}+v\nabla^2 u_z.$$

........(2.09)

$$\frac{\partial b_r}{\partial t}=\frac{B_\theta}{r}\frac{\partial u_r}{\partial \theta}+B_z\frac{\partial u_r}{\partial z}-\Omega\frac{\partial b_r}{\partial \theta}. \qquad \text{........(2.10)}$$

$$\frac{\partial b_\theta}{\partial t}=\frac{B_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+B_z\frac{\partial u_\theta}{\partial z}+\left(\frac{B_\theta}{r}-DB_\theta\right)u_r+(rD\Omega)b_r-\Omega\frac{\partial b_\theta}{\partial \theta}. \qquad \text{....(2.11)}$$

$$\frac{\partial b_z}{\partial t}=\frac{B_\theta}{r}\frac{\partial u_z}{\partial \theta}+B_z\frac{\partial u_z}{\partial z}-u_r DB_z-\Omega\frac{\partial b_z}{\partial \theta}. \qquad \text{........(2.12)}$$

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0. \qquad \text{........(2.13)}$$

$$\frac{\partial b_r}{\partial r}+\frac{b_r}{r}+\frac{1}{r}\frac{\partial b_\theta}{\partial \theta}+\frac{\partial b_z}{\partial z}=0. \qquad \text{........(2.14)}$$

Analyzing the disturbance into normal modes, we seek solutions of the fore going equations whose dependence on t, θ and Z is given by

$$f = f(r)e^{i(\sigma t + m\theta + kz)}, \qquad \text{.........(2.15)}$$

where σ is a constant (which may be complex), m is an integer (+ve, -ve, 0) and k is the wave number of the disturbance in the Z direction.

Let u_r, u_θ etc now denote the amplitudes of the various perturbations whose $t - \theta - Z$ dependence is given by (2.15). Equations (2.07) – (2.12) becomes

$$i\omega u_r - 2\Omega u_\theta - \frac{1}{\rho}\left\{\frac{imB_\theta}{r}(1+\beta)b_r - \frac{2B_\theta b_\theta}{r}\right\}$$

$$= -\frac{1}{\rho}\frac{\partial(\partial p)}{\partial r} + \nu\left\{Lu_r - \frac{2im}{r^2}u_\theta\right\}. \qquad \text{..... (2.16)}$$

$$i\omega u_\theta + (2\Omega + rD\Omega)u_r - \frac{1}{\rho}\left\{\frac{imB_\theta}{r}(1+\beta)b_\theta + b_r DB_\theta + \frac{B_\theta b_r}{r}\right\}$$

$$= -\frac{im(\partial p)}{\rho r} + \nu\left\{Lu_\theta + \frac{2im}{r^2}u_r\right\}. \qquad \text{........(2.17)}$$

$$i\omega u_z - \frac{1}{\rho}\left\{\frac{imB_\theta}{r}(1+\beta)b_z + b_r DB_z\right\} = -\frac{iK}{\rho}(\partial p) + \nu\left\{\left(L + \frac{1}{r^2}\right)u_z\right\}.$$

$$\text{........ (2.18)}$$

$$i\omega b_r = \frac{imB_\theta}{r}(1+\beta)u_r, \qquad \text{........(2.19)}$$

$$i\omega b_\theta = \frac{i\omega B_\theta}{r}(1+\beta)u_\theta + \left(\frac{B_\theta}{r} - DB_\theta\right)u_r + (rD\Omega)b_r. \qquad \text{........(2.20)}$$

$$i\omega b_z = \frac{i\omega B_\theta}{r}(1+\beta)u_z - u_r DB_z. \qquad \text{........(2.21)}$$

where

$$\omega = \sigma + m\Omega\,,$$

$$L = DD^* - \frac{m^2}{r^2} - k^2\,,$$

$$D^* = D + \frac{1}{r}$$

and

$$\beta = \frac{rkB_z}{mB_\theta}.$$

Let

$$\left.\begin{aligned} u_r &= i\omega\xi_r \\ u_\theta &= i\omega\xi_\theta - \xi_r(rD\Omega) \\ u_z &= i\omega\xi_z \end{aligned}\right] \qquad \text{........(2.22)}$$

In the light of Lagrangian displacement vector $\underline{\xi}$ in (2.22), equations (2.19) – (2.21) becomes

$$\left.\begin{aligned} b_r &= \frac{imB_\theta}{r}(1+\beta)\xi_r \\ b_\theta &= \frac{imB_\theta}{r}(1+\beta)\xi_\theta + \left(\frac{B_\theta}{r} - DB_\theta\right)\xi_r \\ b_z &= \frac{imB_\theta}{r}(1+\beta)\xi_z - (DB_z)\xi_r \end{aligned}\right] \qquad \text{........(2.23)}$$

In terms of Lagrangian vector $\underline{\xi}$ equations (2.16) – (2.18) becomes

$$\left(\omega^2 p - \omega iA - B\right)\underline{\xi} + F = 0, \qquad \text{........(2.24)}$$

where

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$iA = \begin{bmatrix} -\nu L & -2\left(\Omega - \frac{im\nu}{r^2}\right) & 0 \\ -2\left(\Omega - \frac{im\nu}{r^2}\right) & -\nu L & 0 \\ 0 & 0 & -\nu L - \frac{\nu}{r} \end{bmatrix}$$

$$B = \begin{bmatrix} \begin{matrix} m^2 v_\theta^2(1+\beta)^2 - 2im\nu D\Omega \\ + rD\Omega^2 - rH_r \end{matrix} & 2imv_\theta^2(1+\beta) & 0 \\ -2imv_\theta^2(1+\beta) & m^2 v_\theta^2(1+\beta)^2 & 0 \\ 0 & 0 & m^2 v_\theta^2(1+\beta)^2 \end{bmatrix}$$

and

$$v_\theta^2 = \frac{B_\theta^2}{\rho r^2}.$$

From (2.01) and (2.22) lead to

$$\frac{\partial \xi_r}{\partial r}+\frac{\xi_r}{r}+\frac{im\xi_\theta}{r}+iK\xi_z=0. \qquad \text{.........(2.25)}$$

Clearly the induced magnetic field $\underline{b}$ with component (b_r, b_θ, b_z) given by (2.23) satisfies

$\nabla.\underline{b}=0$ by virtue of (2.25).

The boundary conditions

$u_r = 0$ at $r = a$

$u_r = 0$ at $r = b$ becomes on using (2.22)

$u_r = 0$ $\xi_r(a)=\xi_r(b)=0.$

Since the ω in the rotating frame is constant, the equation of the form

$$\omega^2\underline{\xi}-\omega iA\underline{\xi}-B\underline{\xi}+F=0,$$

is convenient for, the case of uniform rotation.

For the case of differential rotation it is more convenient to rewrite (2.24) as

$$\sigma^2\underline{\xi}-2\sigma ic\underline{\xi}-G\underline{\xi}+F=0, \qquad \text{.........(2.26)}$$

where

$$c\underline{\xi}=\left(\frac{1}{2}A+im\Omega p\right)\underline{\xi}$$

and

$$G\underline{\xi} = \left(m\Omega iA + B - m^2\Omega^2 p\right)\underline{\xi}$$

Equations (2.24) and (2.26) constitute the basic equation of the stability analysis.

If the fluid is non viscous, the non Hermitian operators will be reduced to Hermitian operators which have been obtained earlier by Ganguly and Gupta [12]

3-3 STABILITY OF AXISYMMETRIC PERTURBATION (m=0)

In this iA, C and G in the characteristic equation becomes

$$iA = \begin{bmatrix} -\nu L & -2\Omega & 0 \\ 2\Omega & -\nu L & 0 \\ 0 & 0 & -\nu\left(L - \frac{1}{r^2}\right) \end{bmatrix}, \quad C = \frac{1}{2}A$$

And

$$G = B = \begin{bmatrix} K^2 r^2 v_z^2 + rD\Omega^2 - rH_r & 2iKrv_\theta v_z & 0 \\ -2iKrv_\theta v_z & K^2 r^2 v_z^2 & 0 \\ 0 & 0 & K^2 r^2 v_z^2 \end{bmatrix}$$

3-4 INNER PRODUCT

Here operator iA and B are not Hermitian. This property mention that stability is not always straight forward like the one carried by K. Ganguly and A.S. Gupta [12] in the case of inviscid fluid.

Inner product defined for any operator B as

$$(\xi, B\xi) = \int_a^b \bar{\underline{\xi}}.B\underline{\xi} r dr. \qquad \text{........(4.01)}$$

With the above definition of inner product

$$(\xi, F) = \int_a^b \left[-\bar{\xi}_r p' - \bar{\xi}_\theta \left(\frac{imp}{r} \right) - \bar{\xi}_z (ikp) \right] r dr$$

$$= \left[-r\bar{\xi}_r p \right]_a^b + \int_a^b D\left(r\bar{\xi}_r \right) P - \int_a^b \bar{\xi}_\theta \left(\frac{imP}{r} \right) r dr - \int_a^b \bar{\xi}_z (iKP) r dr$$

$$= \left[-r\xi_r p \right]_a^b + \int_a^b p \left[\frac{1}{r} \frac{d}{dr} (r\xi_r) + \frac{im\xi_\theta}{r} + ik\xi_z \right] r dr$$

$$= 0 \text{ [By dint of (2.25)]}, \qquad \text{........(4.02)}$$

where prime and over bar denote the differentiation with respect to r and complex conjugate respectively. It can be shown from (2.26) that the operators p, C and G are defined on the entire Hilbert space H, map H into itself and satisfy

$$(p\xi, \eta) = (\xi, p\eta) \text{ for all } \xi, \eta \text{ in H.}$$

Thus by the Hellinger and Toeplitz theorem, these operators are all bounded on H. It follows from (2.26) and (4.02) that

$$\left(\underline{\xi}, \left(\sigma^2 p - 2\sigma iC - G \right) \underline{\xi} \right) = 0$$

$$\sigma^2 \left(\underline{\xi}, \underline{\xi} \right) - 2\sigma (\xi, iC\xi) - \left(\underline{\xi}, G\underline{\xi} \right) = 0, \qquad \text{........(4.03)}$$

where

$$I_1 = (\underline{\xi}, \underline{\xi}),$$

$$(\underline{\xi}, iC\underline{\xi}) = I_2 + iI_3$$

and

$$(\underline{\xi}, G\underline{\xi}) = I_4 + iI_5 .$$

Here I_1, I_2, I_3, I_4 and I_5 are real.

Again

$$I_2 = \int_a^b \left[S(\xi_r^2 + \xi_\theta^2 + \xi_z^2) + \frac{2m\nu R_1}{r^2} - \frac{\nu}{2r}\xi_z^2 \right] r dr, \qquad \text{........(4.04)}$$

$$I_3 = \int_a^b [2\Omega R_1] r dr , \qquad \text{........(4.05)}$$

$$I_4 = \int_a^b \left[X(\xi_r^2 + \xi_\theta^2 + \xi_z^2) + (rD\Omega^2 - rH_r)\xi_r^2 - \frac{m\Omega\nu}{r^2}\xi_z^2 + 2yR_1 \right] r dr, \qquad \text{....(4.06)}$$

and

$$I_5 = \int_a^b [4m\,\Omega^2 R_1 - 2m\,\nu D\Omega\xi_r^2] r dr , \qquad \text{........(4.07)}$$

where

$$S = -\frac{1}{2}\nu L - m\Omega$$

$$X = m^2 v_\theta^2 (1+\beta)^2 - m\Omega\nu L - m^2\Omega^2,$$

$$Y = 2mv_\theta^2 (1+\beta) + \frac{2m^2\Omega\nu}{r^2}$$

and

$$R_1 = \text{Im}(\xi_r \bar{\xi}_\theta).$$

Thus we find that (4.03) is a quadratic equation in σ with complex coefficient. Its roots are

$$\sigma = \frac{2\left(\underline{\xi}, \mathrm{i}c\,\underline{\xi}\right) \pm \sqrt{4\left(\underline{\xi}, \mathrm{i}c\,\underline{\xi}\right)^2 + 4\left(\underline{\xi}, \underline{\xi}\right)\left(\underline{\xi}, G\,\underline{\xi}\right)}}{2\left(\underline{\xi}, \underline{\xi}\right)}$$

$$= \frac{I_2 + iI_3 \pm \sqrt{(I_2 + iI_3)^2 + I_1(I_4 + iI_5)}}{I_1} = \frac{I_2 + iI_3 \pm \sqrt{I_6 + iI_7}}{I_1}$$

$$= \frac{I_2 + iI_3 \pm \lambda^{\frac{1}{2}}\left(\cos\frac{1}{2}\theta + i\operatorname{Sin}\frac{1}{2}\theta\right)}{I_1}.$$

Separating the real and imaginary parts we obtain

$$\sigma_r = \frac{I_2 \pm \lambda^{\frac{1}{2}}\cos\frac{1}{2}\theta}{I_1}$$

and

$$\sigma_i = \frac{I_3 \pm \lambda^{\frac{1}{2}}\operatorname{Sin}\frac{1}{2}\theta}{I_1},$$

where

$$\lambda^2 = I_6^2 + I_7^2,$$

$$\theta = \tan^{-1}\frac{I_7}{I_6},$$

$$I_6 = I_2^2 - I_3^2 + I_1 I_4$$

and

$$I_7 = 2I_2 I_3 + I_1 I_5.$$

3-5 CONCLUSION

For the propagation of unstable mode $\sigma_i \neq 0$

$$\sigma_i = \frac{I_3 \pm \{\lambda^{\frac{1}{2}}(-1 \pm \sqrt{1+y^2})\}/\mu}{I_1}$$

where

$$\mu = \sqrt{y^2 + (1+\sqrt{1+y^2})^2} > 0.$$

The following are the possible values of σ_i

$$\sigma_{i_1} = \frac{1}{I_1}\left[I_3 + \{\lambda^{\frac{1}{2}}(-1+\sqrt{1+y^2})\}/\mu\right]$$

$$\sigma_{i_2} = \frac{1}{I_1}\left[I_3 + \{\lambda^{\frac{1}{2}}(-1-\sqrt{1+y^2})\}/\mu\right]$$

$$\sigma_{i_3} = \frac{1}{I_1}\left[I_3 - \{\lambda^{\frac{1}{2}}(-1+\sqrt{1+y^2})\}/\mu\right]$$

$$\sigma_{i_4} = \frac{1}{I_1}\left[I_3 - \{\lambda^{\frac{1}{2}}(-1-\sqrt{1+y^2})\}/\mu\right]$$

From (4.04) it is clear that $I_3 \rangle 0$

(i) σ_{i_1} and σ_{i_4} are always > 0, accordingly stable mode will be propagated in these cases.

(ii) Propagation will be stable, oscillatory or unstable according as

$$I_3 \gtreqless \lambda^{\frac{1}{2}}(1+\sqrt{1+y^2})/\mu .$$

(iii) Propagation will be stable, oscillatory or unstable or unstable according as

$$I_3 \gtreqless \lambda^{\frac{1}{2}}(-1+\sqrt{1+y^2})/\mu .$$

The object of this paper is to investigate the stability of a steady MHD flow with helical magnetic field. Non axisymmetric disturbances of this flow generally twist the magnetic lines of force and produce an intimate coupling between rotational and hydromagnetic effects. On the other hand axisymmetric disturbances only bent but not twist the magnetic lines of force. When the swirl velocity is large the hydromagnetic effects becomes small. These considerations provide the motivation for studying the stability of non axisymmetric disturbances of MHD helical flow which is likely to be of importance in problems of controlled thermonuclear reaction.

CHAPTER - IV

MHD FLOW OF A HIGHLY CONDUCTING VISCOUS FLUID BETWEEN NON CONDUCTING PARALLEL WALLS

4-1 INTRODUCTION

If an electrically conducting continuum be moving in the magnetic field, the motion of the continuum is changed by the influence of the magnetic field and the magnetic field is also perturbed by the motion of the continuum; one affects the other and vice versa. The motion of the conducting fluid across the magnetic field generates electric currents which changes the magnetic field and the action of the magnetic field on those currents gives rise to mechanical force which modify the flow of the fluid. Stoke's (Schlichting) [14] studied the problem of an incompressible viscous fluid flow produced by the oscillation of a plane solid wall first. Kerczekk and Davis [16] performed the linear stability analysis of the Stoke's layers on the oscillating surface. Panton [15] obtained the transient solution for the flow due to the oscillating plane. Erdagon [18] derived the analytic solutions for the flow produced by the small oscillation wall for small and large times by Laplace Transform Method.

Sinha and Chaudhury [13] discussed the periodic movement of the plate in the non-conducting fluid. Das and Sengupta [17] discussed the unsteady flow of a conducting viscous fluid through a straight tube. In this problem, our main aim is to investigate the effects of a transverse magnetic field on the steady incompressible highly conducting viscous fluid between two non-conducting parallel planes.

4-2 MATHEMATICAL FORMULATION

Let us consider a highly conducting viscous fluid flowing between two non-conducting parallel planes in presence of an uniform transverse magnetic field perpendicular to the planes. The fluid assumes to be incompressible and the motion is steady. As the fluid particles tend to bind themselves to the magnetic field, it is obvious that the field will in some way inhibit the motion of the fluid. The motion will produce tension in the lines of force, but because of the finite conductivity they can revert to their initial positions.

Let $\overline{H}_0 = (0,0,H_0)$ be the uniform magnetic field. And $\overline{q}_0 = (0, v(z), 0)$ be the velocity of the fluid. The Lorentz force on the moving stream will oppose the motion together with the viscous forces.

The motion of the fluid across the field will induce electric current at right angles to the velocity $\overline{q}$. The motion of the fluid will produce a perturbation field intensity $(0,h(z),o)$. Then the total field is $\overline{H}=\overline{H}_0+\overline{h}$ and satisfies $\nabla.\overline{H}=0$. We assume the pressure in the field to be of the form $P=p_0(y)+p_1(z)$. The term $p_0(y)$ gives rise to a pressure gradient $-\frac{\partial p_0}{\partial y}$ in the direction of motion, $p_1(z)$ is ascribable to hydrostatic causes. The governing equations of motion are as follows

$$\rho(\overline{q}.\nabla)\overline{q}=-\nabla(p_0+p_1)-\rho g\underline{k}+\frac{\mu}{4\pi}(\nabla_\wedge\overline{h})_\wedge\overline{H}+\rho\upsilon\nabla^2\overline{q}. \qquad \text{.........(2.01)}$$

$$\nabla_\wedge(\overline{q}_\wedge\overline{H})+\eta\nabla^2\overline{H}=0. \qquad \text{.........(2.02)}$$

$$\nabla.\overline{q}=0,\ \nabla.\overline{H}=0. \qquad \text{.........(2.03)}$$

$$\nabla_\wedge\overline{H}=4H\overline{J} \qquad \text{.........(2.04)}$$

where

$$\overline{J}=\sigma\left[\overline{E}+\mu\overline{q}_\wedge\overline{H}\right]$$

and

$$\eta=\frac{1}{4\pi\mu\sigma}.$$

Equation (2.01) becomes

$$0=-\frac{\partial p_0}{\partial y}\underline{j}-\frac{\partial p}{\partial z}\underline{k}-\rho g\underline{k}+\frac{\mu}{4\pi}\left\{H_0h'\underline{j}-hh'\underline{k}+\rho r\frac{d^2v}{dz^2}\underline{j}\right\}$$

Equating corresponding coefficient we get

$$\rho v\frac{d^2v}{dz^2}+\frac{\mu H_0h'}{4\pi}=\frac{dp_0}{dy} \qquad \text{.........(2.05)}$$

and

$$\frac{\mu}{4\pi}hh' + \frac{dp_1}{dz} + \rho g = 0 \qquad \text{........(2.06)}$$

Equation (2.02) becomes

$$H_0\frac{dv}{dz} + \eta\frac{d^2h}{dz^2} = 0. \qquad \text{........(2.07)}$$

Here equations(2.05), (2.06) and (2.07) are the characteristics equations of motions. Equation (2.05) implies that for steady laminar flow, the pressure gradient in the direction of motion is constant throughout the liquid.

From (2.06)

$$p_1 = c - \rho gz - \frac{\mu}{8\pi}h^2. \qquad \text{........(2.08)}$$

From (2.07) we get

$$\frac{dh}{dz} + 4\pi\mu\sigma H_0 v = C_2 \qquad \text{........(2.09)}$$

From $\nabla_\wedge \overline{H} = 4\pi\overline{J}$ and $\overline{J} = \sigma\left[\overline{E} + \mu\overline{q}_\wedge \overline{H}\right]$ we get

$$-h'i = 4\pi(j_1\underline{i} + j_2\underline{j} + j_3\underline{k}) = 4\pi\sigma\left[E_1\underline{i} + E_2\underline{j} + E_3\underline{k} + H_0\mu vi\right]$$

implies

$$-h'(z) = 4\pi j_1 = 4\pi\sigma(E_1 + \mu H_0 v) \qquad \text{........(2.10)}$$

$$j_2 = \sigma E_2 = 0 \qquad \text{........(2.11)}$$

$$j_3 = \sigma E_3 = 0 \qquad \text{........(2.12)}$$

From (2.10) we get

$$\frac{dh}{dz} + 4\pi\sigma\mu H_0 \upsilon = -4\pi\sigma E_1 = C_2 \text{ using } (2.09)$$

$$\therefore -E_1 = \frac{1}{4\pi\sigma}\frac{dh}{dz} + \mu H_0 v = \frac{c_2}{4\pi\sigma} \qquad \text{........(2.13)}$$

From (2.05) and (2.13) we get

$$\rho\upsilon\frac{d^2\upsilon}{dz^2} - \sigma\mu^2 H_0^2 v = \sigma\mu H_0 E_1 - p = c \text{ (Constant)}$$

or

$$\frac{d^2v}{dz^2} - \frac{\sigma\mu^2 H_0^2}{\rho\upsilon}\upsilon = \frac{c}{\rho\upsilon}$$

$$\therefore v = A\operatorname{Cosh}\frac{M}{L}z + B\operatorname{Sinh}\frac{M}{L}z - \frac{c}{\sigma\mu^2 H_0^2}$$

where $M = \mu H_0 L\sqrt{\frac{\sigma}{\rho\upsilon}}$ Hartmann number.

Boundary conditions are $v = 0$ at $z = \pm L$ implies

$$v = \frac{c(\operatorname{Cosh}\frac{M}{L}z - \operatorname{Cosh}M)}{\sigma\mu^2 H_0^2 \operatorname{Cosh}M}$$

Now $\int_{-L}^{h} J_1 dz = 0$ implies

$$E_1 = \frac{p(\operatorname{Sinh}M - M\operatorname{Cosh}M)}{\sigma\mu H_0 \operatorname{Sinh}M}$$

$$\therefore c = \sigma\mu H_0 E_1 - p = -\frac{-pM\operatorname{Cosh}M}{\operatorname{Sinh}M}$$

$$\therefore v = \frac{pM(\operatorname{Cosh}M - \operatorname{Cosh}\frac{M}{L}z)}{\sigma\mu^2 H_0^2 \operatorname{Sinh}M}$$

and

$$\bar{v} = \frac{1}{2L}\int_{-L}^{L} v\,dz = \frac{p(M\operatorname{Cosh}M - \operatorname{Sinh}M)}{\sigma\mu^2 H_0^2 \operatorname{Sinh}M}$$

So

$$\frac{v}{\bar{v}} = \frac{M(\operatorname{Cosh}M - \operatorname{Cosh}\frac{M}{L}z)}{M\operatorname{Cosh}M - \operatorname{Sinh}M}$$

For a weak magnetic field $M = 0$

$$\therefore v = \frac{3}{2}\bar{v}(1 - \frac{z^2}{L^2})$$

Using the value of v in (2.07) and integrating we get

$$h = \frac{4\pi pL\operatorname{Sinh}\frac{M}{L}z}{\mu H_0 \operatorname{Sinh}M} + a_1 z + a_2$$

at $z = \pm L$, $h = 0$ implies

$$h = \frac{4\pi pL}{\mu H_0}\left(\frac{\operatorname{Sinh}\frac{M}{L}z}{\operatorname{Sinh}M} - \frac{z}{L}\right)$$

For a weak magnetic field $h = 0$.

Table

$\frac{V}{\upsilon}$											
M = 10	1.111	1.110	1.110	1.110	1.108	1.103	1.090	1.056	.960	.702	0
M = 8	1.142	1.142	1.140	1.138	1.133	1.122	1.096	1.039	.912	.629	0
M = 6	1.194	1.193	1.189	1.181	1.167	1.140	1.091	1.001	.838	.541	0
M = 4	1.284	1.280	1.268	1.244	1.207	1.149	1.062	.930	.733	.439	0
M = 2	1.417	1.407	1.376	1.322	1.244	1.139	1.002	.827	.608	.336	0
M= 0	1.5	1.485	1.440	1.365	1.260	1.125	.960	.765	.510	.285	0
Z →	0	$\pm\frac{L}{10}$	$\pm\frac{2L}{10}$	$\pm\frac{3L}{10}$	$\pm\frac{4L}{10}$	$\pm\frac{5L}{10}$	$\pm\frac{6L}{10}$	$\pm\frac{7L}{10}$	$\pm\frac{8L}{10}$	$\pm\frac{9L}{10}$	$\pm$L

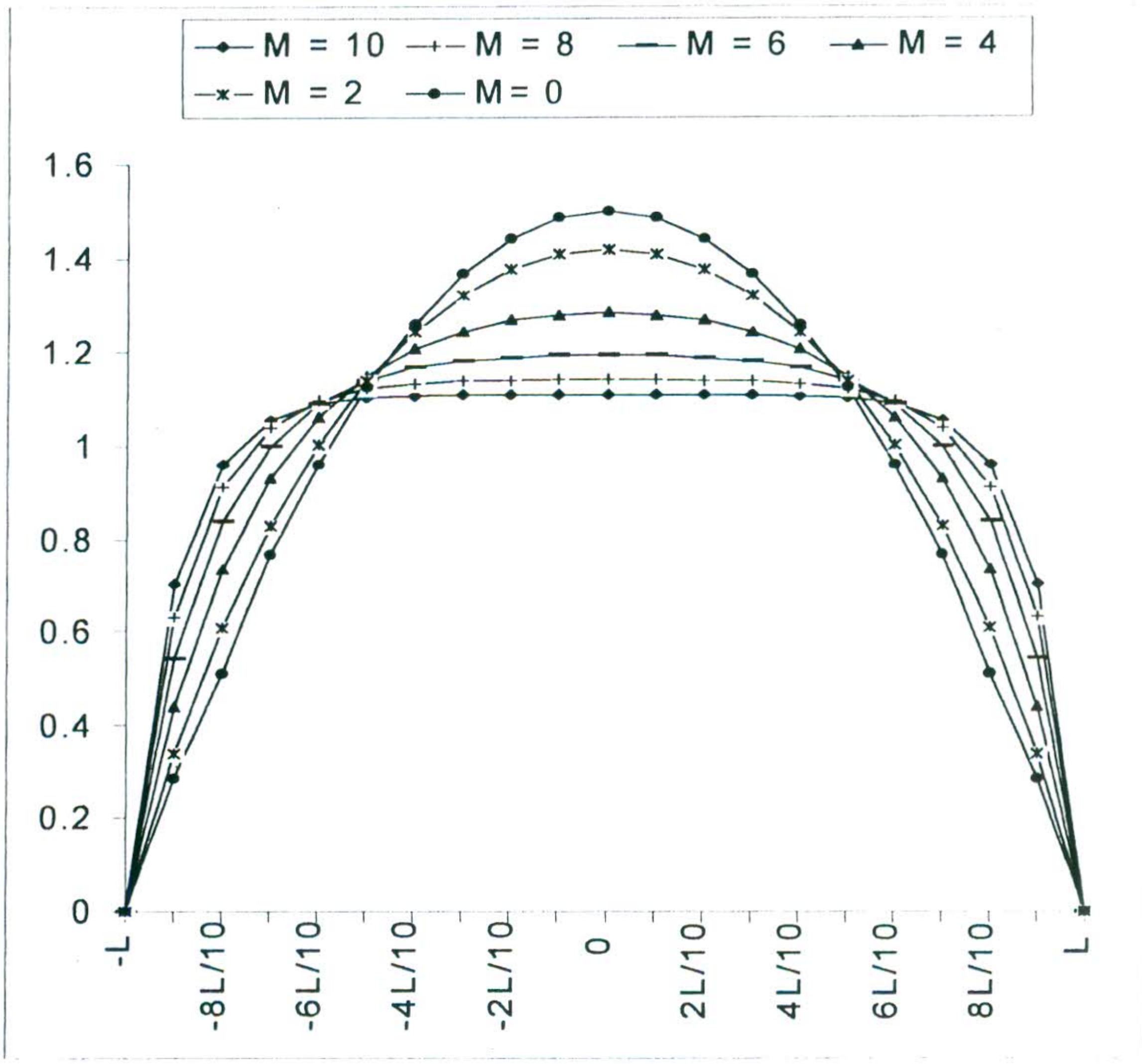

Fig 1: Gives a Sketch of the Velocity Profiles for Various Values of Hartmann Number.

4-3 CONCLUSION

When M = 0 then $v = \frac{3}{2}\bar{v}(1 - \frac{z^2}{L^2})$ which shows that the velocity profile between the parallel plates in absence of magnetic field is parabolic. When magnetic field is present $(M \neq 0)$ then the velocity profile is also parabolic but less concavity. Accordingly Hartman number increases, the concavity of velocity profile decreases.

CHAPTER - V

UNSTEADY FLOW OF VISCO-ELASTIC RIVLIN-ERICKSEN FLUID WITH TRANSIENT PRESSURE GRADIENT THROUGH A UNIFORM CIRCULAR CYLINDER

5-1 INTRODUCTION

The fluids which exhibit the elastic property of solids and viscous property of fluids are adequate in nature and the relevant fluids generate visco-elastic fluid mechanics. This discipline is termed as Rheolog of continua. This types of fluids are also called non-Newtonian fluids or visco-elastic fluids. The hydro-dynamic flow of different fluids of viscous and inviscid types has been presented in the standard informative books of Goldstein [19], Lamb [20], Milne-Thompson [21], Pai [22], Landau [23], Batchlor [24], Curle and Davis [25] and others. The corresponding development of hydromagnetic flow of viscous problems have been given in the standard books of Alfven [26], Cowling [27], Chandrasekhar [5], Ferraro and Plumpton [28], Jefery [29], Cabannes [30]. In recent years, some problems associated with the visco-elastic liquids and fluids have been considered by Sengupta [31]. He suggested some empirical general models. Sengupta, SK. Kundu and S.K. Misra [42] studied this problem through rectangular channel. In this paper, an attempt has been made to study the flow of visco-elastic conducting fluid of Rivlin-Ericksen type flowing in a circular cylinder in presence of a uniform transverse magnetic field.

5-2 MATHEMATICAL ANALYSIS

We consider the cylinder $x'^2 + y'^2 = a^2$, $z' = 0$, z' as a boundary wall and z' axis is the axis of the cylinder, direction of motion. Let us assume $W'(x', y', t')$ be the axial velocity of the fluid. B_0 is the uniform transverse magnetic field perpendicular to the velocity W' is applied to the fluid. Under these assumptions, the governing equations for the present problem are as follows

$$\frac{\partial W'}{\partial t'} = \frac{-1}{\rho}\frac{\partial p'}{\partial z'} + \nu\left(1 + \mu_1{}'\frac{\partial}{\partial t'}\right)\nabla^2 W' - \frac{B_0^2}{\rho_1} W' \quad \text{........(2.01)}$$

$$\frac{\partial W'}{\partial z'} = 0. \quad \text{........(2.02)}$$

If we introduce the non-dimensional quantities

$$(x', y', z') = a(x, y, z),$$

$$W = \frac{v}{a} W,$$

$$p' = \frac{\rho v^2}{a^2} p,$$

$$t' = \frac{a^2}{v} t,$$

$$\mu_1' = \frac{a^2}{v} \mu$$

and

$$M = aB_0 \sqrt{\frac{\sigma}{\rho v}} \text{ (Hartmann number).}$$

Equation (2.01) becomes

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + (1 + \mu_1 \frac{\partial}{\partial t}) \nabla^2 W - M^2 W \qquad \text{........(2.03)}$$

with boundary condition W = 0 on the surface $x^2 + y^2 = 1$.

Here, $\nabla^2 W = \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}$

$$= \frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}$$

$$= \frac{1}{r}\frac{d}{dr}\left(r\frac{dW}{dr}\right) + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}$$

The symmetric about the origin requires W is a function of r only, so that $\frac{\partial^2 W}{\partial\theta^2} = 0$.

Thus

$$\nabla^2 W = \frac{1}{r}\frac{d}{dr}\left(r\frac{dW}{dr}\right).$$

If we introduce transient pressure gradient $-pe^{-\omega t}$ and consider the transient axisymmetric velocity W is of the form $f(r)e^{-\omega t}$, then equation (2.03) in cylinderical co-ordinate system becomes

$$-\omega f = p + (1-\mu_0\omega)\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)\right\} - M^2 f$$

Subject to the boundary condition $f(1) = 0$

or

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + K^2 f = N,\ f(1) = 0 \qquad \text{........(2.04)}$$

where $K^2 = \frac{\omega - M^2}{1-\mu_1\omega}$ and $N = \frac{-P}{1-\mu_0\omega}$

To solve equation (2.04) we apply Laplace transformation as

$$L\{f(r)\} = \int_0^\alpha e^{-sr} f(r)dr = F(s). \qquad \text{........(2.05)}$$

Then we get from equation (2.04)

$$F(S) = \frac{N}{K^2 S} + \frac{C}{\sqrt{S^2 + K^2}}$$

$$\therefore f(r) = L^{-1}F(s) = \frac{N}{K^2} + CJ_0(Kr)$$

where $J_0(Kr)$ is the Bessel function of order zero,

$$f(1) = 0 \text{ implies } C = \frac{-N}{K^2 J_0(K)}$$

$$\therefore f(r) = \frac{N}{K^2}\left\{1 - \frac{J_0(Kr)}{J_0(K)}\right\}. \quad \text{.........(2.06)}$$

We now consider the case where K is very small so that we approximate K as

$$J_0(kr) = 1 - \frac{K^2 r^2}{4}$$

and

$$J_0(k) = 1 - \frac{K^2}{4}$$

Substituting these in (2.06) we obtain

$$f(r) = \frac{p(1-r^2)}{4-(1+\mu_1)\omega + M^2}$$

$$\therefore W = f(r)e^{-\omega t} = \frac{P(1-r^2)e^{-\omega t}}{4-(1+\mu_1)\omega + M^2} \quad \text{.........(2.07)}$$

5-3 DEDUCTIONS

Case-I : putting $\mu_1 = 0$ in the equation (2.07), we shall obtain the solution for a purely viscous fluid which is given below

$$W = \frac{p(1-r^2)e^{-\omega t}}{4-\omega+M^2}$$

Case-II : When the magnetic field is absent (M=0), the velocity is obtained in the form

$$W = \frac{p(1-r^2)e^{-\omega t}}{4-(1+\mu_1)\omega}$$

5-4 NUMERICAL CALCULATIONS

For numerical calculation of the velocity profile for visco-elastic fluid the following data are considered in (2.07)

$$\mu_1 = 0.05,\ \omega = 10,\ p = 1,\ r = 0.75$$

$$\therefore W = \frac{.4375}{M^2-6.5}e^{-10t}$$

In case-II for ordinary viscous fluid, $\mu_1 = 0$, $\omega = 10$, $p = 1$, $r = 0.75$ are considered.

$$\therefore W = \frac{.4375e^{-10t}}{M^2-6}$$

For different values of M, the results of W are shown in Table-1 and Table-2 respectively.

		Table-1 $\mu_1 = 0.5$		Table-2 $\mu_1 = 0$	
t	e^{-10t}	W		W	
		M=3.10	M=3.10	M=3.15	M=3.15
0	1	.1407	.1212	.1115	.1278
.05	.6065	.0853	.0735	.0676	.0775
.10	.3679	.0517	.0446	.0410	.0470
.15	.2231	.0314	.0270	.0249	.0285
.20	.1353	.0190	.0164	.0151	.0173
.25	.0821	.0115	.0099	.0091	.0104
.30	.0498	.0070	.0060	.0055	.0063
.35	.0302	.0042	.0037	.0034	.0038
.40	.0183	.0026	.0022	.0020	.0023
.45	.0111	.0015	.0013	.0012	.0014
.50	.0067	.0009	.0008	.0007	.0008
.55	.0041	.0006	.0005	.0004	.0005
.60	.0025	.0003	.0003	.0003	.0003
.65	.0015	.0002	.0002	.0002	.0002
.70	.0009	.0001	.0001	.0001	.0001
.75	.0005	0	0	0	0

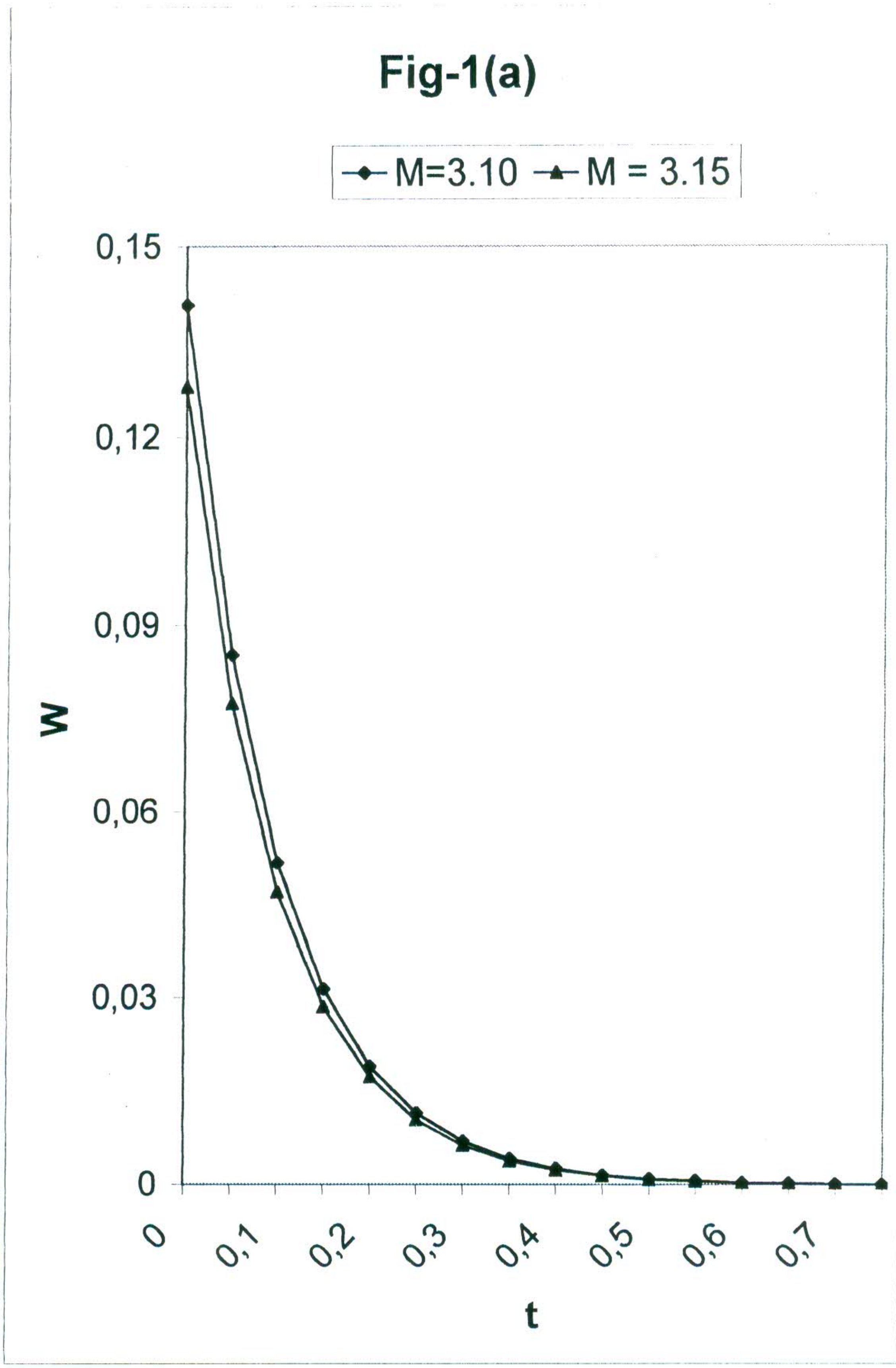

Fig 1(a): Shows the Variation of Velocity of Rivlin Erickesen Fluid with Time for Different Values of Hartmann Number (M).

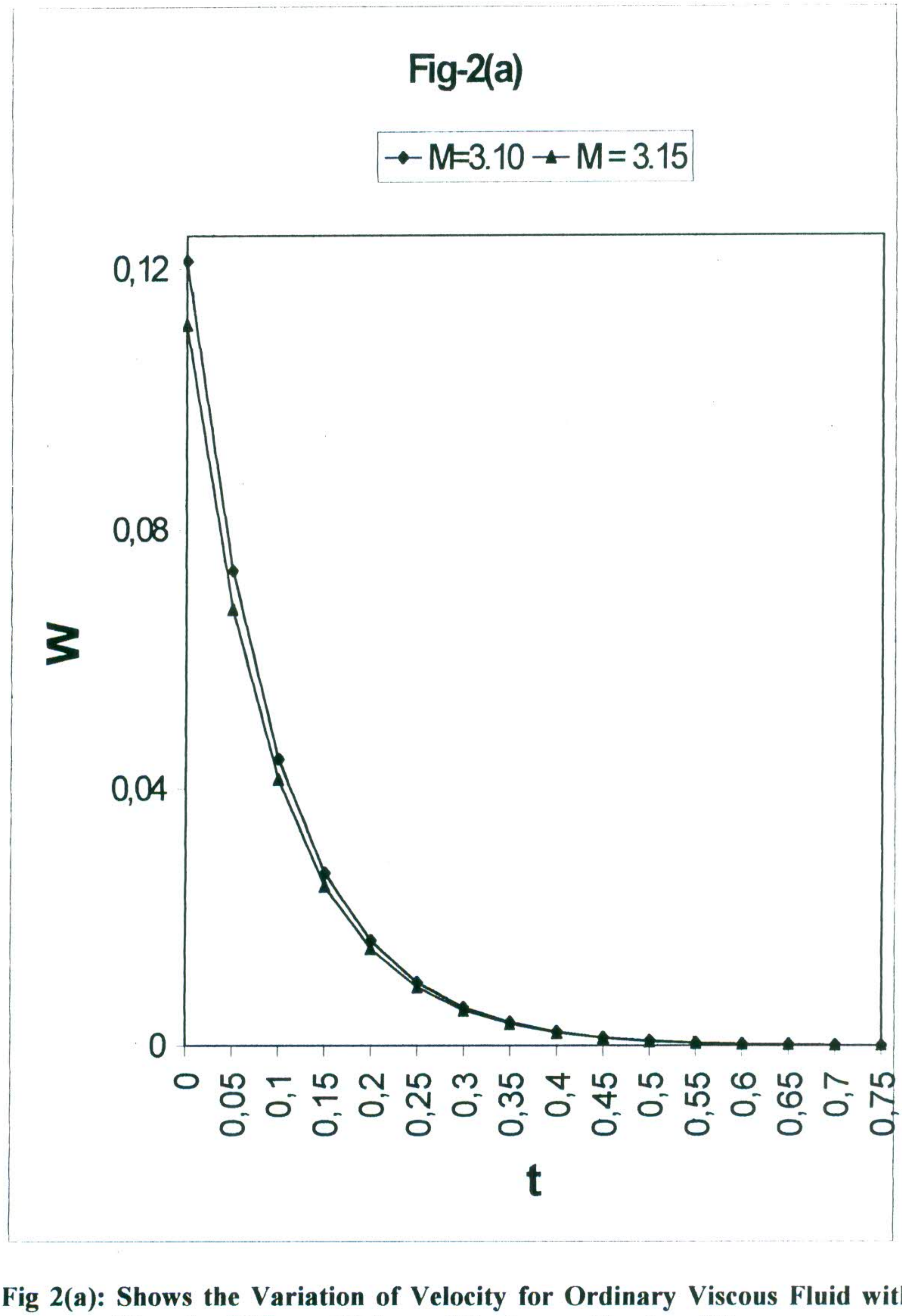

Fig 2(a): Shows the Variation of Velocity for Ordinary Viscous Fluid with Time for Different Values of Hartmann Number (M).

5-5 CONCLUSION

Figure 1 (a): shows that the velocity decreases due to increase of magnetic field that is the velocity profile is stopped by the influence of strong magnetic field.

Figure 2(a): also shows that the velocity decreases due to increase of magnetic field, but velocity is higher in Rivlin-Ericksen fluid than ordinary viscous fluid.

CHAPTER - VI

UNSTEADY MHD FLOW OF VISCO-ELASITIC INCOMPRESSIBLE FLUID BETWEEN TWO CONCENTRIC CYLINDERS WITH TRANSIENT PRESSURE GRADIENT

6-1 INTRODUCTION

The hydromagnetic flow of visco-elastic fluid, such as organic compounds, oils etc play a considerable role in technological and engineering fields. The basic development of magneto-hydromagnetics have been considered by Cowling [27]. Gradual development in the realm of hydrodynamics has been found to be satisfactory by Sengupta and Ghose [31] and Sengupta and Bhattacharya [32]. The problems of viscous motion between two parallel plates under the action of initially applied body force studied by Carslaw and Jaegar [33]. Sengupta and Roymahapatra [34] studied the flow of two inmiscible visco-elastic Maxwell fluid through rectangular channel with transient pressure gradient.

The motion of visco-elastic Maxell fluid subject to a uniform or periodic body force was studied by Pal and Sengupta [35]. Unsteady hydromagnetic flow of two immiscible visco-elastic Oldroyd fluid between two parallel plates has been studied by Ghose and Sengupta [36]. In this paper we have studied the unsteady MHD flow of visco-elastic Rivlin-Ericksen fluid with transient pressure gradient through concentric cylinders.

6-2 MATHEMATICAL FORMULATION

The Rheological equations relating to the stress-tensor τ_{ij} and the rate of Strain tensor e_{ij} for the slow motion of visco-elastic Rivlin-Ericksen type fluid are

$$\left.\begin{aligned} \tau_{ij} &= p'\delta_{ij} + \tau'_{ij} \\ \tau'_{ij} &= 2\mu'(1+\lambda'\frac{\partial}{\partial t})e_{ij} \\ e_{ij} &= \frac{1}{2}(u_{ij} + u_{ji}) \end{aligned}\right\} \qquad \text{.........(2.01)}$$

Where τ_{ij} is the deviatoric stress tensor, p' is the pressure, μ' is the viscous parameter and λ' is the strain retardation time.

We consider the mass of electrically conducting visco-elastic Rivlin-Ericksen type fluid between two concentric cylinders. The fluid initially at rest. A transverse uniform magnetic field B_0 has been applied to the fluid. The effect due to induced magnetic and the perturbation of the magnetic field is neglected. The equations of slow motion of a conducting visco-elastic Rivlin-Ericksen type fluid in three dimensional form become

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu(1+\lambda'\frac{\partial}{\partial t'})(\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}) - \frac{\sigma B_0^2}{\rho}u'. \qquad \text{........(2.02)}$$

$$\frac{\partial u'}{\partial x'} = 0. \qquad \text{........(2.03)}$$

We are now going to put the equation (2.02) in a non-dimensional form by setting

$$u = \frac{a}{\nu}u',$$

$$p = \frac{a^2}{\rho\nu^2}p',$$

$$t = \frac{\nu}{a^2}t',$$

$$\lambda = \frac{\nu}{a^2}\lambda',$$

$$(x, y, z) = \frac{1}{a}(x', y', z')$$

and

$$M = aB_0\sqrt{\frac{\sigma}{\rho\nu}} \text{ (Hartmann Number).}$$

Thus the governing equation in the non-dimensional form is

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+(1+\lambda\frac{\partial}{\partial t})(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})-M^2u \qquad \text{........(2.04)}$$

The boundary conditions of the problem in non-dimensional form are u = 0 on the surface

$$y^2+z^2=1$$

and

$$y^2+z^2=\frac{b^2}{a^2}\ (b>a)$$

If we introduce transient pressure gradient $-pe^{-\omega t}$ and consider the transient axisymmetric velocity is of the form $F(y,z)e^{-\omega t}$ then equation (2.04) becomes

$$-\omega F=p+(1-\lambda\omega)(\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 F}{\partial z^2})-M^2F$$

or

$$\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 F}{\partial z^2}+k^2F=N \qquad \text{........(2.05)}$$

where

$$k^2=\frac{\omega-M^2}{1-\lambda\omega}$$

and

$$N=\frac{-p}{1-\lambda\omega}$$

$$\therefore F=\frac{N}{k^2}+A\mathrm{Cos}kS+B\mathrm{Sin}kS$$

where $S=y\mathrm{Cos}\alpha+z\mathrm{Sin}\alpha$, with boundary conditions F=0 when S=1 and $S = \frac{b}{a}$.

According to boundary conditions

$$F = \frac{N}{k^2}\left\{\frac{\operatorname{Cos}\frac{k(b-a)}{2a} - \operatorname{Cos}k\left(S - \frac{a+b}{2a}\right)}{\operatorname{Cos}\frac{k(b-a)}{2a}}\right\} \qquad \ldots\ldots..(2.06)$$

We now consider the case where k is very small so that we approximate k as

$$\operatorname{Cos}\frac{k(b-a)}{2a} = 1 - \frac{k^2(b-a)^2}{8a^2}$$

and

$$\operatorname{Cos}k\left(S - \frac{a+b}{2a}\right) = 1 - \frac{1}{2}k^2\left(s - \frac{a+b}{2a}\right)^2.$$

Substituting these in (2.06) we obtain

$$F = \frac{4pa(S-1)(b-aS)}{8a^2(1-\lambda\omega) - (\omega - M^2)(b-a)^2}. \qquad \ldots\ldots..(2.07)$$

For numerical calculation of the velocity profile, $\lambda = 0.05$, $\omega = 10$, $\rho = 1$, $S = 1.5$, $a = 1$ and $b = 2$

$$u = \frac{e^{-10t}}{M^2 - 6}.$$

t	e^{-10t}	Table-1 $\lambda = 0.05$ W		Table-2 $\lambda = 0$ W	
		M=3.10	M=3.15	M=3.10	M=3.15
0	1	.2770	.2549	.1314	.1262
.05	.6065	.1680	.1546	.0797	.0765
.10	.3679	.1019	.0938	.0483	.0464
.15	.2231	.0618	.0568	.0293	.0281
.20	.1353	.0375	.0345	.0178	.0171
.25	.0821	.0227	.0209	.0108	.0103
.30	.0498	.0138	.0127	.0065	.0063
.35	.0302	.0083	.0077	.0039	.0038
.40	.0183	.0050	.0046	.0024	.0029
.45	.0111	.0030	.0028	.0014	.0014
.50	.0067	.0018	.0017	.0009	.0008
.55	.0041	.0011	.0010	.0005	.0005
.60	.0025	.0007	.0006	.0003	.0003
.65	.0015	.0004	.0004	.0002	.0002
.70	.0009	.0002	.0002	.0001	.0001
.75	.0005	.0001	.0001	0	0
.80	.0003	0	0	0	0

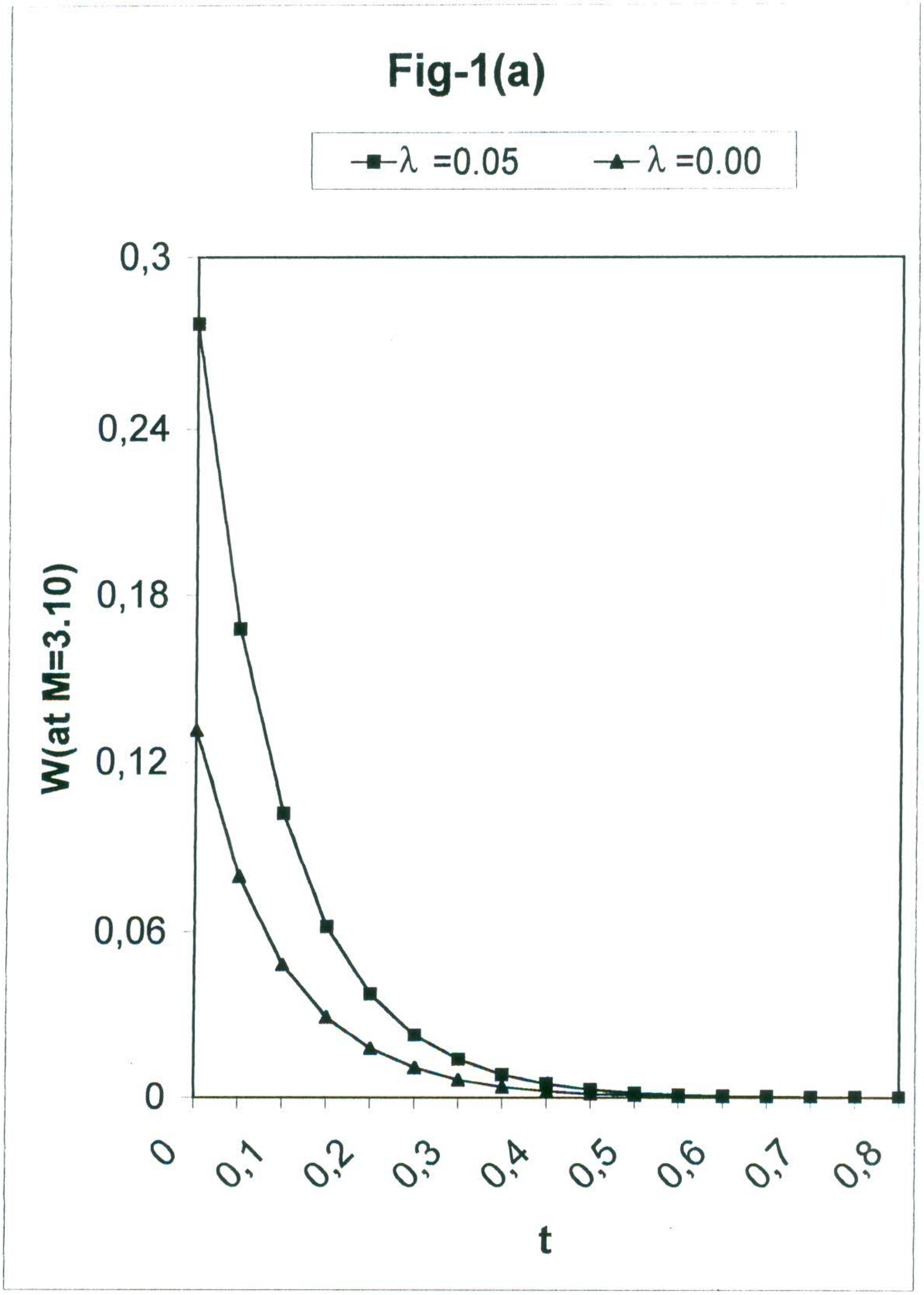

Fig 1(a): Shows the Variation of Rvilin Ericksen Fluid Velocity and Ordinary Viscous Fluid Velocity with Time When M = 3.10.

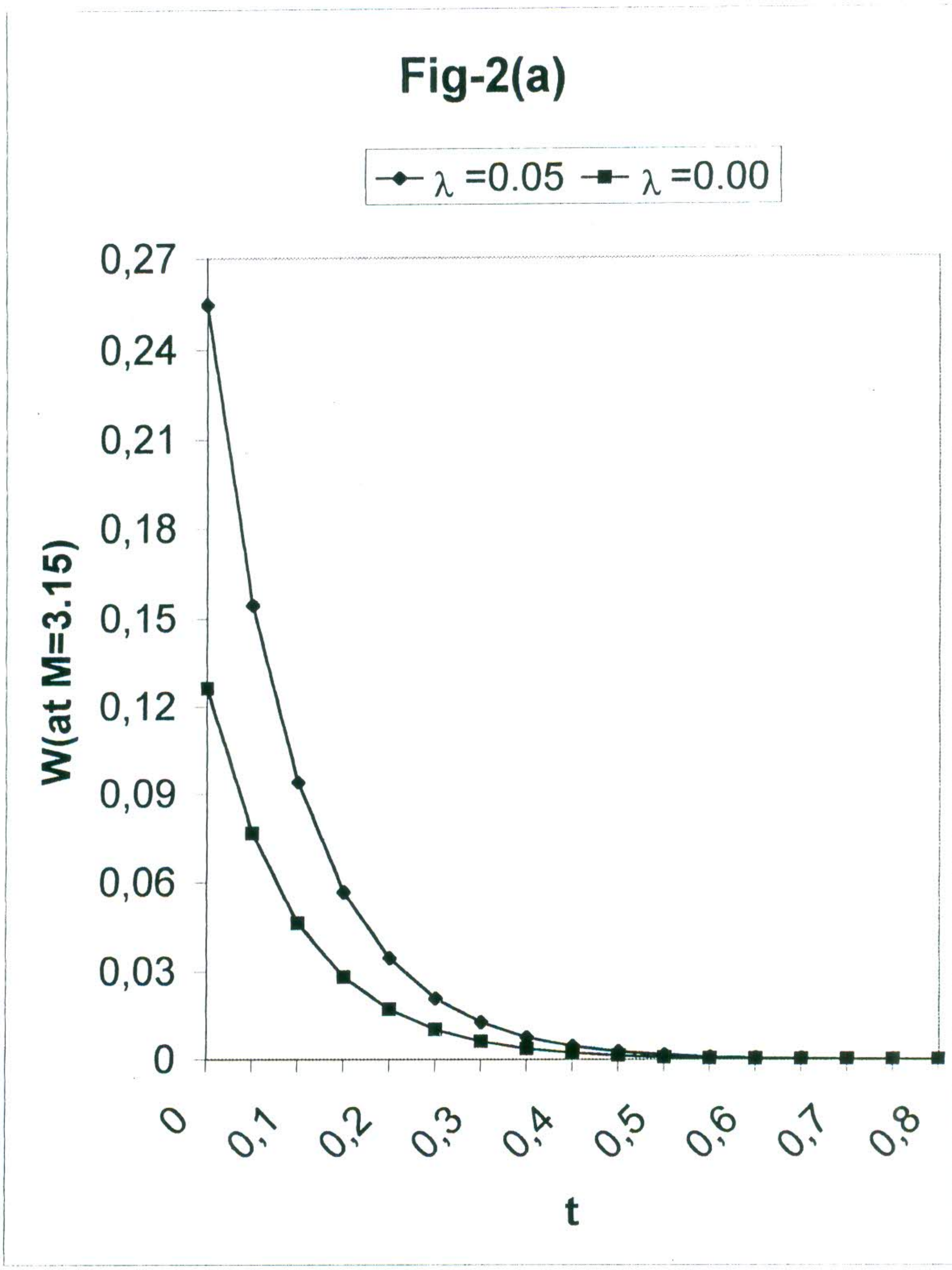

Fig 2(a): Shows the Variation of Rvilin Ericksen Fluid Velocity and Ordinary Viscous Fluid Velocity with Time When M = 3.16.

6-3 CONCLUSION

Fig1(a) and Fig 2(a) shows the following

(1) Velocity decreases due to increase of magnetic field.

(2) In Visco-elastic Rivlin-Ericksen fluid, the velocity is greater than in ordinary viscous fluid in the presence of same magnetic field.

(3) Velocities of both kind of fluids are die out as $t \rightarrow \propto$

CHAPTER - VII

UNSTEADY MHD FLOW OF VISCO-ELASTIC OLDROYD FLUID BETWEEN TWO CONCENTRIC CYLINDER

7-1 INTRODUCTION

In elastic materials the stress depends on the strain only. So we can say that the elastic materials have memory, i.e., it is capable of recognizing its original shapes. Some fluids have no memory. But there are some fluids like soap solution, polymers, which have some elastic properties, besides having fluid properties. Such type of fluids are called non-Newtonian fluids or visco-elastic fluids. The basic development of hydrodynamic flow has been presented in the work of Lamb[20] and hydromagnetic problems have been considered by Cowling [27], Carslaw and Jaeger [33] studied the basic problem of viscous motion between two parallel plates under the action initially applied body force. Das [37] studied hydromagnetic flow of viscous conducting fluid through a circular cylinder. Sengupta and Roymahpatra [34] studied visco-elastic Maxwell fluid through rectangular channel with transient pressure gradient. The hydromagnetic flow of two immiscible visco-elastic Walter liquids between two inclined parallel plates has been investigated by Chakrabarty and Sengupta [39] Ghose and Sengupta [36] studied the unsteady hydromagnetic flow of two immiscible visco-elastic Oldroyd fluid between two parallel plates. In this paper we have considered unsteady MHD flow of visco-elastic fluid of Oldroyd type with time varying body force through a concentric cylinders.

7-2 MATHEMATICAL ANALYSIS

We consider the cylinders $x'^2 + y'^2 = a^2$ and $x'^2 + y'^2 = b^2$ (a>b) are the boundary walls and z'axis is the axis of cylinders, the direction of motion. Let us consider $W'(x', y', t')$ be the velocity of the fluid. Assuming the motion to be slow and neglecting pressure gradient, the equation of motion of a conducting visco-elastic Oldroyd fluid in the present studies becomes

$$(1+\lambda_1' \frac{\partial}{\partial t'})(\frac{\partial W'}{\partial t'}) = (1+\lambda_1' \frac{\partial}{\partial t'}) \times (t') + v(1+\lambda_2' \frac{\partial}{\partial t'})\nabla^2 W' - \frac{\alpha B_0^2}{\rho}(1+\lambda_1' \frac{\partial}{\partial t'})W' \qquad \text{.........(2.01)}$$

$$\frac{\partial W'}{\partial z'} = 0 \qquad \text{.........(2.02)}$$

where ν the kinematic coefficient of viscosity, ρ is the constant density of the fluid, B_0 is the uniform magnetic field, σ is the electrical conductivity, λ_1' and λ_2' $(\lambda_1' > \lambda_2' > 0)$ are stress relaxation time and rate of strain retardation time respectively. We are now going to put the equation (2.01) in a non-dimensional form by setting

$$W = \frac{a}{\nu} W',$$

$$t = \frac{\nu}{a^2} t',$$

$$\lambda = \frac{\nu}{a^2} \lambda',$$

$$(x, y, z) = \frac{1}{a}(x', y', z')$$

and

$$M = aB_0 \sqrt{\frac{\sigma}{\rho\nu}} \text{ (Hartman number).}$$

Thus the governing equation (2.01) in the non-dimensional form is

$$(1+\lambda_1 \frac{\partial}{\partial t})\frac{\partial W}{\partial t} = (1+\lambda_1 \frac{\partial}{\partial t}) \times (t) + (1+\lambda_2 \frac{\partial}{\partial t})\nabla^2 W - M^2(1+\lambda_1 \frac{\partial}{\partial t})W \quad \text{.........(2.03)}$$

The boundary conditions of the problem in non-dimensional form are

(i) $W = 0$ on the surface $x^2 + y^2 = 1$ and

(ii) $W = 0$ on the surface $x^2 + y^2 = \frac{b^2}{a^2}$

7-3 SOLUTION OF THE PROBLEM

Case-I: Fluid motion due to transient body force; Firstly, we have consider a transient body force $X_0 e^{-\omega t}$ is applied to the fluid and assume velocity is of the form

$$W = V(x, y)e^{-wt},$$

then the (2.03) becomes

$$\nabla^2 V + k^2 V = N. \qquad \text{........(3.01)}$$

where

$$K^2 = \frac{(1-\lambda_1\omega)(\omega - M^2)}{(1-\lambda_2\omega)} \text{ and}$$

$$N = -\frac{(1-\lambda_1\omega)}{(1-\lambda_2\omega)} X_0$$

with boundary conditions V = 0 on the surface

$$\left.\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + y^2 &= \frac{b^2}{a^2} \end{aligned}\right] \qquad \text{. ....(3.02)}$$

Solution of (3.01) with boundary condition (3.02) is

$$V = \frac{X_0}{M^2 - \omega}\left[1 - \frac{\text{Cos}k\left(\frac{a+b}{2a} - S\right)}{\text{Cos}k\left(\frac{a-b}{2a}\right)}\right]$$

where $S = x\cos\alpha + y\sin\alpha$

and

$$W = \frac{X_0}{M^2 - \omega} e^{-\omega t} \left[1 - \frac{\operatorname{Cosk}\left(\frac{a+b}{2a} - S \right)}{\operatorname{Cosk}\left(\frac{a-b}{2a} \right)} \right] \qquad \text{.........(3.03)}$$

We now consider the case where k is very small so that we approximate k as

$$\operatorname{Cosk}\left(\frac{b-a}{2a} \right) = 1 - \frac{k^2 (b-a)^2}{8a^2}$$

and

$$\operatorname{Cosk}\left(\frac{a+b}{2a} - S \right) = 1 - \frac{k^2 \left(\frac{a+b}{2a} - S \right)^2}{2}$$

substituting these in (3.03) we obtain

$$W = \frac{4aX_0 (1 - \lambda_1 \omega)(S-1)(b-aS)}{(1 - \lambda_2 \omega)\{8a^2 - k^2 (a-b)^2} e^{-\omega t} \qquad \text{.........(3.04)}$$

Case II: Fluid motion due to periodic body force. In this case, we assume that the body force $X_0 e^{i\omega t}$ is applied to the fluid and consider the velocity is of the form $V(x,y) e^{i\omega t}$, the equation (2.03) becomes

$$\nabla^2 V - k_1^2 V = -N_1 \qquad \text{.........(3.05)}$$

where

$$k_1^2 = \frac{-M^2 (1 + \lambda_1 \lambda_2 \omega^2) + \omega^2 (\lambda_1 - \lambda_2)}{1 + \lambda_2^2 \omega^2} + \frac{i\omega\{(\lambda_1 - \lambda_2) M^2 + (1 + \lambda_1 \lambda_2 \omega^2)\}}{1 + \lambda_2^2 \omega^2}$$

$$= Rw^{i\theta} (\text{Say})$$

and

$$N_1 = \frac{1+\lambda_1\lambda_2\omega^2}{1+\lambda_2^2\omega^2}X_0 - \frac{i\omega(\lambda_1-\lambda_2)}{1+\lambda_2^2\omega^2}X_0$$

$$= R_1 e^{-i\theta_1}\,(\text{Say})$$

The solution of (3.05) subject to the boundary condition (3.02) is

$$V = \frac{R_1}{R}e^{-i(\theta_1+\theta)}\left[1-\frac{\operatorname{Cos}k_1\left(\frac{a+b}{2a}-S\right)}{\operatorname{Cos}k_1\left(\frac{a-b}{2a}\right)}\right] \qquad \ldots\ldots..(3.06)$$

$$W = \frac{1}{\sqrt{M^4+\omega^2}}\left[\left\{1-\frac{\operatorname{Cosh}(1-S)A\operatorname{Cos}\left(\frac{b}{a}S\right)B}{\operatorname{Cosh}(a-b)A+\operatorname{Cos}(a-b)B}\right\}\operatorname{Cos}(\omega t-\theta_1-\theta)\right.$$

$$\left.+\left\{\frac{\operatorname{Sinh}(1-S)A\operatorname{Sin}\left(\frac{b}{a}-S\right)B}{\operatorname{Cosh}(a-b)A+\operatorname{Cos}(a-b)B}\right\}\operatorname{Sin}(\omega t-\theta_1-\theta)\right] \qquad \ldots\ldots..(3.07)$$

only real part taken.

where

$$\tan\theta = \frac{\omega\{(\lambda_1-\lambda_2)M^2+(1+\lambda_1\lambda_2\omega^2)\}}{-M^2(1+\lambda_1\lambda_2\omega^2)+\omega^2(\lambda_1-\lambda_2)}$$

and

$$\tan\theta_1 = \frac{\omega(\lambda_1-\lambda_2)}{1+\lambda_1\lambda_2\omega^2}$$

$$R^2 = \frac{(M^4+\omega^2)(1+\lambda_1^2\omega^2)}{1+\lambda_2^2\omega^2},$$

$$R_1^2 = \frac{1+\lambda_1^2\omega^2}{1+\lambda_2^2\omega^2} X_0$$

and

$$A = \sqrt{R}\,\mathrm{Cos}\frac{1}{2}\theta, B = \sqrt{R}\,\mathrm{Sin}\frac{1}{2}\theta.$$

Then, it may be concluded that the velocity of Oldroyd fluid is a periodic function of t with period $\frac{2\pi}{n}$. It is also observed that the velocity depends on the radii a and b.

7-4 THE FLOW PATTERN IN ABSENCE OF MAGNETIC FIELD

We are now going to find out the velocity component due to flow in the absence of magnetic field.

Case I: Transient body force and absence of magnetic field. we put M = 0 in equation (3.03) we get

$$W = \frac{X_0}{\omega}\left[\frac{\mathrm{Cosk}\left(\frac{a+b}{2a} - S\right)}{\mathrm{Cosk}\left(\frac{a-b}{2a}\right)} - 1\right] e^{-\omega t}.$$

This velocity is transient and depends on the radii a, b and dies out as $t \to \infty$.

Case II: Periodic body force and absence of magnetic field:

If we put M = 0 in equation (3.07) then we get

$$W = \frac{1}{\omega}\left[\left\{1 - \frac{\text{Cosh}(1-S)A\,\text{Cos}\left(\frac{b}{a} - S\right)B}{\text{Cosh}(a-b)A + \text{Cos}(a-b)B}\right\}\text{Cos}(\omega t - \theta_1 - \theta)\right.$$

$$\left. + \left\{\frac{\text{Sinh}(1-S)A\,\text{Sin}\left(\frac{b}{a} - S\right)B}{\text{Cosh}(a-b)A + \text{Cos}(a-b)B}\right\}\text{Sin}(\omega t - \theta_1 - \theta)\right]$$

where

$$R^2 = \frac{1 + (1 + \lambda_1^2\omega^2)}{1 + \omega^2},$$

$$A = \sqrt{R}\,\text{Cos}\frac{1}{2}\theta,$$

$$B = \sqrt{R}\,\text{Sin}\frac{1}{2}\theta$$

and

$$\tan\theta = \frac{(1 + \lambda_1\lambda_2\omega^2)}{\omega(\lambda_1 - \lambda_2)} = -\cot\theta_1 = \tan\left(\frac{\pi}{2} + \theta_1\right)$$

$$\therefore \theta = \frac{\pi}{2} + \theta_1 \text{ and } \tan\theta_1 = \frac{\omega(\lambda_1 - \lambda_2)}{1 + \lambda_1\lambda_2\omega^2}$$

The velocity is periodic with period $\frac{2\pi}{n}$ and depends on radii a and b.

7-5 DISCUSSIONS OF THE STABILITY OF FLOW

Rout-Hurwitz stability criteria for a small disturbing forces is that the deviation is small from the initial condition of motion. In case of application of transient body force $X = X_0 e^{-\omega t}$ the motion of the fluid will be stable if $\omega > 0$.

In our problem, we have the relation

$$K^2 = \frac{(1 - \lambda_1\omega)(\omega - M^2)}{1 - \lambda_2\omega}.$$

This leads to the characteristic equation as

$$\lambda_1\omega^2 - (1 + \lambda_1 M^2 + \lambda_2 k^2)\omega + k^2 + M^2 = 0 \qquad \text{........(5.01)}$$

The roots of the equation are

$$\omega = \frac{1 + \lambda_1 M^2 + \lambda_2 k^2 \pm \sqrt{(1 + \lambda_1 M^2 + \lambda_2 k^2) - 4\lambda_1 (k^2 + M^2)}}{2\lambda_1}$$

Clearly, for stability of transient motion, the solution (3.03) tends to zero or a finite value as $t \rightarrow \infty$. It is possible, if both the roots of equation (5.01) are positive. This yields the condition as

$$(1 + \lambda_1 M^2 + \lambda_2 k^2)^2 - 4\lambda_1 (k^2 + M^2) > 0 \qquad \text{........(5.02)}$$

and

$$1 + \lambda_1 M^2 + \lambda_2 k^2 \pm \sqrt{(1 + \lambda_1 M^2 + \lambda_2 k^2)^2 - 4\lambda_1 (k^2 + M^2)} > 0 \qquad \text{.......(5.03)}$$

Conditions (5.02) and (5.03) implies

$$\left.\begin{array}{l} 0<\omega<\left\{\dfrac{1}{\lambda_1}+M^2\left(1-\dfrac{\lambda_2}{\lambda_1}\right)\right\} \\ \\ M^2<\dfrac{1}{\lambda_2} \\ \\ \lambda_2<\lambda_1 \end{array}\right] \qquad \text{.........(5.04)}$$

Thus, the condition (5.04) ensures the stability of the transient motion.

If $\lambda_2 > \lambda_1$ and $M^2 > \dfrac{1}{\lambda_2}$, then the roots of the characteristic equation (5.01) in ω are complex conjugate. The positive real part indicating the motion is stable with a damped oscillation type.

7-6 NUMERICAL CALCULATION:

For numerical calculation of the velocity profile for visco-elastic Oldroyd fluid, the following data are consider in (3.04)

$\lambda_1 = 0.05, \lambda_2 = 0.005, \omega = 10, a = 2, b = 1, X_0 = 1$ and $S = .75$

t	e^{-10t}	Table-1 W $\lambda_1 = 0.05$ $\lambda_2 = 0.005$		Table-2 W $\lambda_1 = 0$ $\lambda_2 = 0.005$		Table-2 W $\lambda_1 = \lambda_2 = 0$	
		M= 3.10	M= 3.16	M= 3.10	M= 3.16	M= 3.10	M= 3.16
0	1	.0655	.01645	.03332	.03291	.03163	.03126
.05	.6065	.01003	.00997	.02021	.01996	.01918	.01896
.10	.3679	.00609	.00605	.01225	.01211	.01167	.01150
.15	.2231	.00369	.00367	.00749	.00734	.00705	.00697
.20	.1353	.00223	.00222	.00457	.00445	.00428	.00423
.25	.0821	.00136	.00135	.00273	.00270	.00259	.00256
.30	.0498	.00082	.00082	.00166	.00164	.00157	.00155
.35	.0302	.00050	.00050	.00100	.00099	.00095	.00094
.40	.0183	.00030	.00030	.00061	.00060	.00058	.00057
.45	.0111	.00018	.00018	.00037	.00036	.00035	.00034
.50	.0067	.00011	.00011	.00022	.00022	.00021	.00020
.55	.0041	.00006	.00006	.00013	.00013	.00013	.00013
.60	.0025	.00004	.00004	.00008	.00008	.00008	.00008
.65	.0015	.00002	.00002	.00005	.00005	.00005	.00004
.70	.0009	.00001	.00001	.00003	.00003	.00003	.00003
.75	.0005	0	0	.00001	.00001	.00001	.00001
.80	.0003	0	0	0	0	0	0

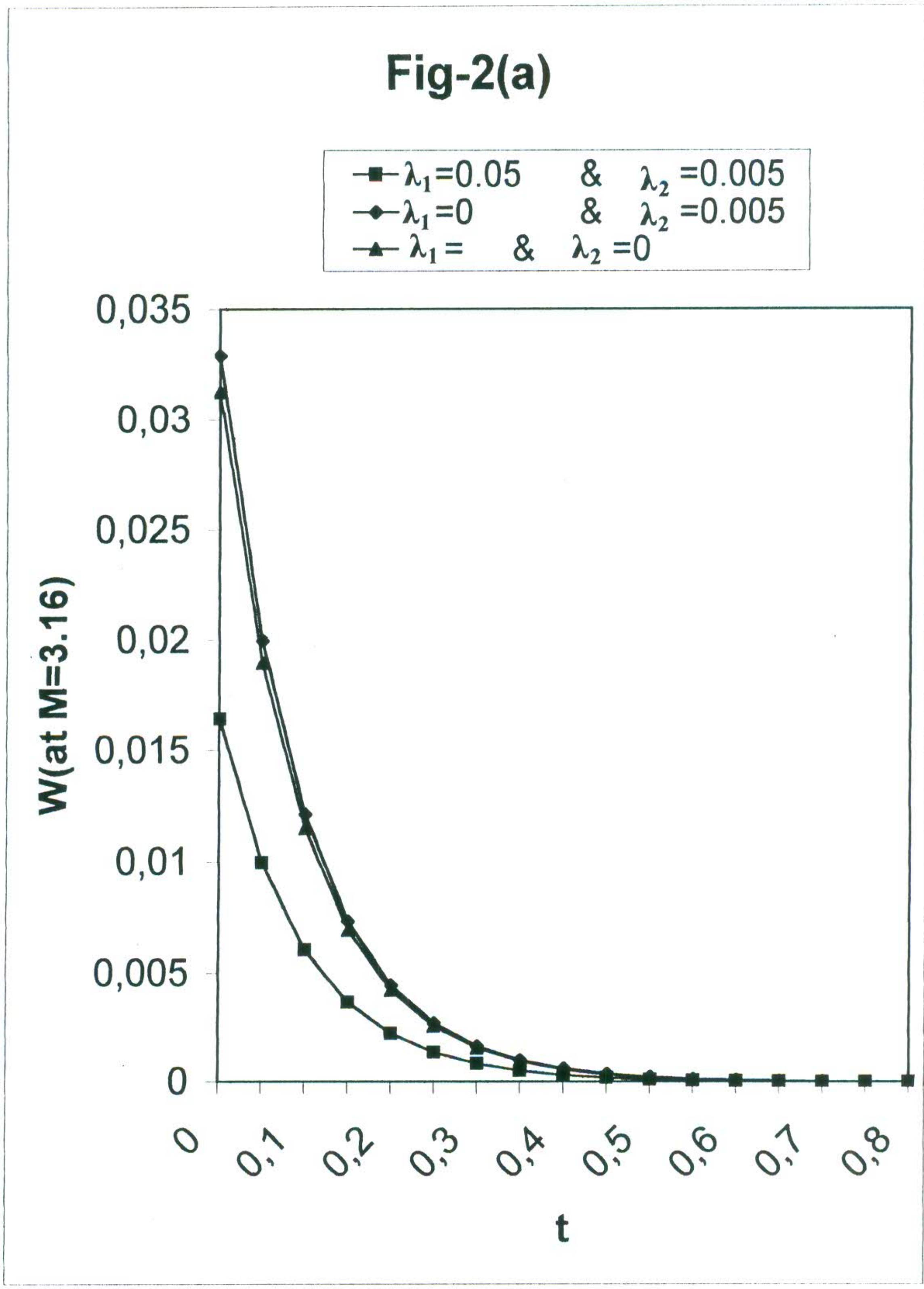

Fig 2(a): Velocity Profiles for Oldroyd Fluid, Rvilin Rricksen Fluid and Ordinary Viscous Fluid at M = 3.16.

7-7 CONCLUSION

Fig1(a) and 2(a) shows the following results:

1. Velocity of Oldroyd is smaller than Rivlin-Ericksen fluid and ordinary viscous fluid.

2. Velocity of Rivlin-Ericksen fluid as greater than Oldroyd fluid and ordinary viscous fluid.

3. Velocity of ordinary viscous fluid is greater than Oldroyd fluid but smaller than Rivlin-Ericksen fluid.

4. Velocity decrease due to increase of magnetic field in the case of all fluids

CHAPTER - VIII

UNSTEADY MHD FLOW OF VISCO-ELASTIC OLDROYD FLUID WITH THE TIME VARYING BODY FORCE THROUGH A RECTANGULAR CHANNEL

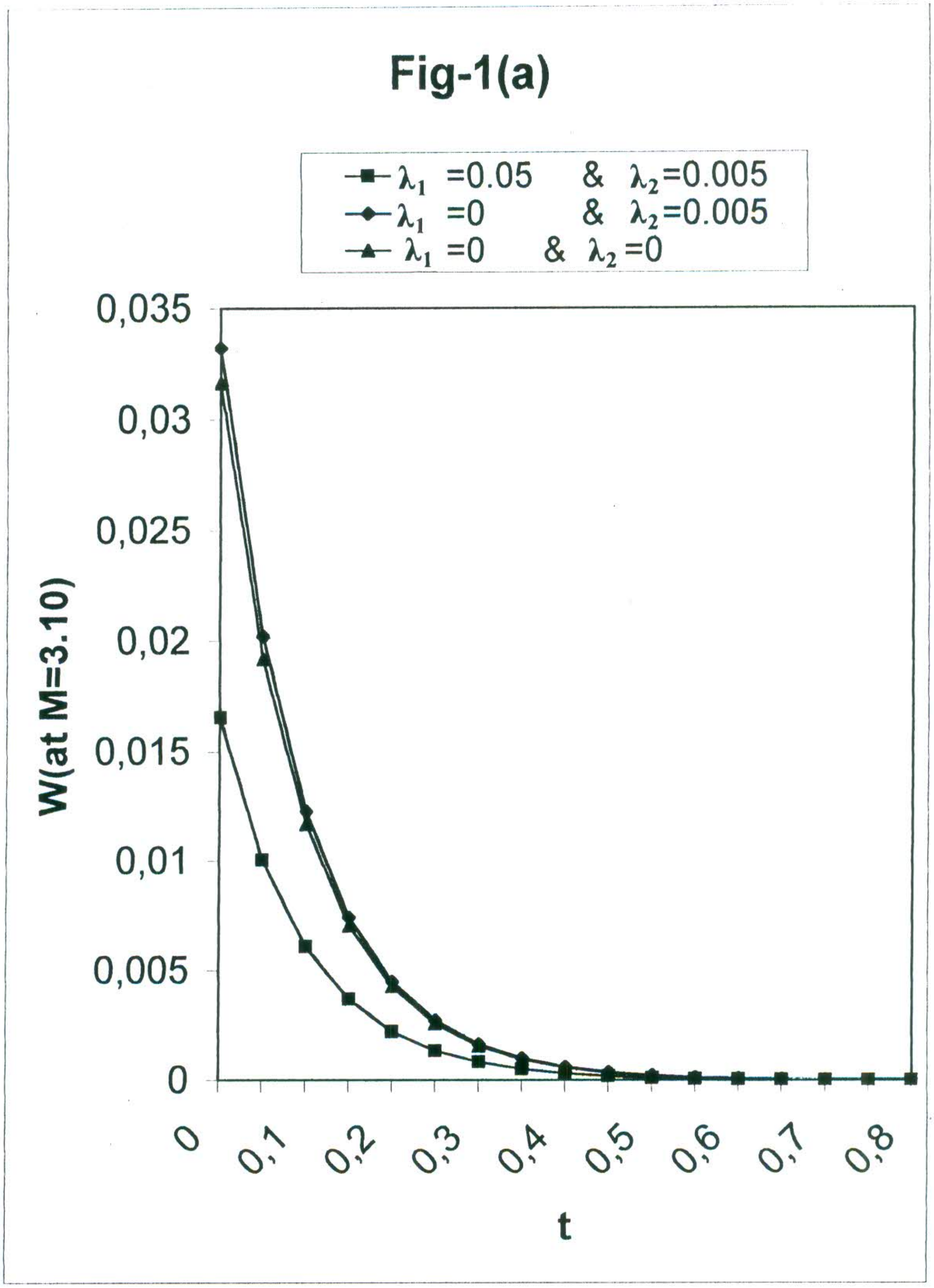

Fig 1(a): Velocity Profiles for Oldroyd Fluid, Rvilin Rricksen Fluid and Ordinary Viscous Fluid at M = 3.10.

8-1 INTRODUCTION

The fluids which exhibit the elasticity property of solids and viscous property of liquids are called visco-elastic fluids or non-Newtonian fluids. The basic development of hydodynamics flow has been presented in the work of Lamb [20] and hydromagnetic problems have considered by Cowling [27]. Carslaw and Jaeger [33] studied the basic problem of viscous motion between two parallel plates under the action initially applied body force, Das[37] studied hydro magnetic flow of viscous conducting fluid through a circular cylinder. The hydromagnetic flow of two immiscible visco-elastic walter liquids between two inclined parallel plates has been investigated by Chakrabarty and Sengupta [38]. Ghose and Sengupta [36] studied the unsteady hydromagnetic flow of two parallel plates. In this paper we have considered unsteady MHD flow of visco-elastic fluid of Oldroyd type with time varying body force through a rectangular channel.

8-2 MATHEMATICAL ANALYSIS

We consider the wall $x^{1} = \pm a$, $y' = \pm b$ and z' axis is taken in the direction of motion. Let us assume $W'(x', y', t')$ be the velocity of the fluid. Assuming the motion to be slow and neglecting pressure gradient and perturbation, the equation of the motion of a conducting visco-elastic Oldroyd fluid in the present studies becomes

$$\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)\left(\frac{\partial W'}{\partial t'}\right) = \left(1+\lambda_1' \frac{\partial}{\partial t'}\right)X(t') + \nu\left(1+\lambda_2' \frac{\partial}{\partial t'}\right)\nabla^2 W' - \frac{\sigma B_0^2}{\rho}\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)W'. \quad \text{.........(2.01)}$$

$$\frac{\partial W'}{\partial z'} = 0. \quad \text{........(2.02)}$$

where ν the kinetic coefficient of viscosity, ρ is the constant density of the fluid, σ is the electrical conductivity, λ_1' and λ_2' $(\lambda_1' > \lambda_2' > 0)$ are the stress relaxation time and rate of strain retardation time respectively. We are now going to put the equation (2.01) and (2.02) in a non-dimensional form by setting

$$W = \frac{a}{\nu} W',$$

$$t = \frac{\nu}{a^2} t',$$

$$\lambda = \frac{\nu}{a^2} \lambda',$$

$$(x, y, z) = \frac{1}{a}(x', y', z')$$

and

$$M = aB_0 \sqrt{\frac{\sigma}{\rho\nu}} \text{ (Hartmann number).}$$

Thus the governing equation (2.01) and (2.02) in the non dimensional form are

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial W}{\partial t}\right) =$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) X(t) + \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \nabla^2 W - M^2 \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) W, \qquad \text{........(2.03)}$$

and

$$\frac{\partial W}{\partial z} = 0. \qquad \text{........(2.04)}$$

The boundary conditions of the problem in non-dimensional form are

(i) $W = 0$ when $x = \pm 1$, $\frac{-b}{a} \le y \le \frac{b}{a}$.

(ii) $W = 0$ when $y = \pm \frac{b}{a}$, $-1 \le x \le 1$.

8-3 SOLUTION OF THE PROBLEM

Case I: Fluid motion due to transient body force; Firstly, we consider a transient body force $X_0 e^{-\omega t}$ is applied to the fluid and assume the velocity is of the form

$$W = F(x, y)e^{-\omega t},$$

then the equation (2.03) becomes

$$\nabla^2 F + k^2 F = -N. \qquad \text{.........(3.01)}$$

where

$$k^2 = \frac{(1-\lambda_1\omega)(\omega - M^2)}{1-\lambda_2\omega} \text{ and}$$

$$N = \frac{1-\lambda_1\omega}{1-\lambda_2\omega} X_0,$$

with boundary conditions $F = 0$

when $x = \pm 1, \ \frac{-b}{a} \le y \le \frac{b}{a}$

and $F = 0$ when $y = \pm\frac{b}{a}, \ -1 \le x \le 1$ ....(3.02).

A solution of (3.01) under the condition (3.02) will be satisfied if

$$\text{Cos} m\frac{b}{a} = 0.$$

$$\therefore m = (2n+1)\frac{\pi a}{2b};\ n = 0,1,2,3,4, \ldots\ldots\ldots\ldots\ldots$$

We construct the solution as the sum of all possible solutions for each value n and therefore

$$F = \sum_{0}^{\infty} f(x)\mathrm{Cos}\,my.$$

Now expressing N as a Fourier series in the interval $\frac{-b}{a} \le y \le \frac{b}{a}$ and equating coefficient of Cosmy, we get

$$\frac{d^2 f}{dx^2} - T^2 f = \frac{-N(-1)^n}{(2n+1)\pi}. \qquad \text{........(3.03)}$$

where

$$T^2 = \left\{(2n+1)\frac{\pi a}{2b}\right\}^2 - \frac{(1-\lambda_1\omega)(\omega - M^2)}{1-\lambda_2\omega}.$$

So, there exist a positive integer n_1 such that $T^2 > 0$ for all $n > n_1$.

Solution of (3.03) is

$$f(x) = \frac{4N(-1)^n}{(2n+1)\pi T^2}\left[1 - \frac{\mathrm{Cosh}\,Tx}{\mathrm{Cosh}\,T}\right] \text{ for } T^2 > 0$$

$$= \frac{4N(-1)^n}{(2n+1)\pi |T|^2}\left[\frac{\mathrm{Cos}\,Tx}{\mathrm{Cos}\,T} - 1\right] \text{ for } T^2 < 0 \qquad \text{.........(3.04)}$$

Hence the solution of (2.03) becomes

$$W = \sum_{n=0}^{n_1} \frac{(-1)^n}{(2n+1)T^2}\left\{\frac{\mathrm{Cos}\,Tx}{\mathrm{Cos}\,T} - 1\right\}\mathrm{Cos}(2n+1)\frac{\pi ay}{2b}e^{-\omega t}\Bigg]$$

$$+ \sum_{n=n_1+1}^{n_1} \frac{(-1)^n}{(2n+1)T^2}\left\{1 - \frac{\mathrm{Cosh}\,Tx}{\mathrm{Cosh}\,T}\right\}\mathrm{Cos}(2n+1)\frac{\pi ay}{2b}e^{-\omega t}\Bigg], \qquad \text{........(3.05)}$$

where n_1 is such that $T^2(n_1) \le 0, T^2(n_1+1) > 0$. However $n_1 = 0$, if $T^2(0) > 0$.

Case II: Fluid motion due to periodic body force. In this case, we assume that the body force $X_0 e^{i\omega t}$ is applied to the fluid and consider the velocity is of the form

$$V(x,y)e^{i\omega t},$$

the equation (2.03) becomes

$$\nabla^2 V + k_1^2 V = -N_1, \qquad \text{.........(3.06)}$$

where

$$k_1^2 = \frac{\omega^2(\lambda_1-\lambda_2) - M^2(1+\lambda_1\lambda_2\omega^2)}{1+\lambda_2^2\omega^2} - \frac{i\omega\{(\lambda_1-\lambda_2)M^2 + (1+\lambda_1\lambda_2\omega^2)\}}{1+\lambda_2^2\omega^2}$$

and

$$N_1 = \frac{1+\lambda_1\lambda_2\omega^2}{1+\lambda_2^2\omega^2} - \frac{i\omega(\lambda_1-\lambda_2)}{1+\lambda_2^2\omega^2}$$

$$= R_1 e^{-i\theta_1}.$$

A solution of equation (3.06) under conditions (3.02) can be taken as $V(x,y) = f(x)\text{Cos}my$, conditions (3.02) will be satisfied if

$$\text{Cos}\frac{mb}{a} = 0$$

or

$$m = (2n+1)\frac{\pi a}{2b}, \quad n = 0,1,2, \ldots\ldots\ldots\ldots .$$

We construct the solution as the sum of all possible solutions for each value of n and expressing N_1 as a Fourier series in the interval $\frac{-b}{a} \le y \le \frac{b}{a}$ and equating coefficient of Cosmy we get

$$\frac{d^2 f}{dx^2} - T^2 f = \frac{-4N_1(-1)^n}{(2n+1)\pi}, \qquad \text{.........(3.07)}$$

where

$$T^2 = m^2 - K_1{}^2 = Re^{i\theta}\,(\text{Say}).$$

Therefore solution of (3.07) is

$$f = \frac{4N_1(-1)^n}{(2n+1)T^2}\left[1 - \frac{\text{CoshTx}}{\text{CoshT}}\right]. \qquad \text{........(3.08)}$$

∴ Periodic solution of (3.03) is

$$W = \frac{4R_1}{\pi R}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\left[\left\{1 - \frac{\text{Cosh}(x+1)A\ \text{Cos}(x-1)B + \text{Cosh}(x-1)A\ \text{Cos}(x+1)B}{\text{Cosh}2A + \text{Cos}2B}\right\}\right.$$

$$\times\left\{\cos(3n+1)\frac{\pi ay}{2b}\right\}\text{Cos}(\omega t - \theta - \theta_1) + \left\{\frac{\text{Sinh}(x+1)A\ \text{Sin}(x-1)B + \text{Sinh}(x-1)A\ \text{Sin}(x+1)B}{\text{Cosh}2A + \text{Cos}2B}\right\}$$

$$\left.\times\left\{\cos(3n+1)\frac{\pi ay}{2b}\right\}\text{Sin}(\omega t - \theta - \theta_1)\right] \qquad \text{........(3.09)}$$

taking real part only, where A and B are dunctions of n, ω, λ_1, λ_2 and M^2.

8-4 DEDUCTIONS

Case I: Putting $\lambda_1 = 0$ in the equation (3.05) we shall obtain visco-elastic Rivlin-Ericksen fluid which is given below

$$W = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)T^2}\left\{1 - \frac{\text{CoshTx}}{\text{CoshT}}\right\}\text{Cos}(2n+1)\frac{\pi ay}{2b}e^{-\omega t}\Bigg],$$

where

$$N = \frac{X_0}{1 - \lambda_2\omega}$$

and

$$T^2 = m^2 - \frac{\omega - M^2}{1-\lambda_2\omega}.$$

Case II: When putting $\lambda_1 = 0$ and $\lambda_2 = 0$ in the equation (3.05), we obtain purely viscous fluid and velocity in this case is

$$W = \frac{4X_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)T^2}\left[1-\frac{\text{Cosh}Tx}{\text{Cosh}T}\right]\text{Cos}(2n+1)\frac{\pi ay}{2b}\times e^{-\omega t},$$

where

$$T^2 = m^2 - \omega + M^2.$$

Case III: When the magnetic field is absent i.e. $M = 0$, the velocity will be in the form

$$W = \frac{4N}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)T^2}\left[1-\frac{\text{Cosh}Tx}{\text{Cosh}T}\right]\text{Cos}(2n+1)\frac{\pi ay}{2b}\times e^{-\omega t},$$

where

$$T^2 = m^2 - \frac{\omega(1-\lambda_1\omega)}{1-\lambda_2\omega}.$$

8-5 DISCUSSIONS OF STABILITY

We study the stability of the flow due to transient body force $X = X_0 e^{-\omega t}$, the motion of the fluid will be stable if $\omega > 0$. It means that the motion of the fluid tends to finite value or zero as t tends to infinity and hence is stable. Both the roots of the characteristic equation in ω are to be positive for motion to be stable.

The presence of a negative root would lead to an unstable transient motion. Also if the roots are complex conjugate with positive real part, the motion will be stable and damped oscillation, while the complex conjugate roots with negative real part will generate instability. In our problem we have the relation

$$k^2 = \frac{(1-\lambda_1\omega)(\omega - M^2)}{1-\lambda_2\omega}.$$

This leads to the characteristic equation as

$$\lambda_1\omega^2 - (1+\lambda_1 M^2 + \lambda_2 k^2)\omega + k^2 + M^2 = 0 \qquad \text{........(5.01)}$$

The roots are

$$\omega = \frac{S \pm \sqrt{S^2 - 4\lambda_1(k^2 + M^2)}}{2\lambda_1},$$

where

$$S = 1 + \lambda_1 M^2 + \lambda_2 k^2.$$

The transient motion will be stable if

$$S^2 - 4\lambda_1(M^2 + k^2) > 0 \qquad \text{........(5.02)}$$

$$S - \sqrt{S^2 - 4\lambda_1(M^2 + k^2)} > 0 \qquad \text{........(5.03)}$$

Relation (5.02) and (5.03) implies

$$\lambda_1 > \lambda_2,$$

$$0 < \omega < \frac{1}{\lambda_1} + M^2\left(1 - \frac{\lambda_2}{\lambda_1}\right) \text{ and}$$

$$M^2 < \frac{1}{\lambda_2}.$$

These are the conditions of stable of visco-elastic oldroyd type fluid.

8-6 NUMERICAL CALCULATION

For numerical calculation of the velocity profile, the following data are considered for equation (4.05)

$\lambda_1 = 0.05, \lambda_2 = 0.005, X_0 = 1, \omega = 10, b = 0.25, a = 0.5, x = 0.75$ and $y = 0.45$

t	Table-1 W $\lambda_1 = 0.05 \quad \lambda_2 = 0.005$				Table-2 W $\lambda_1 = 0 \quad \lambda_2 = 0.005$				Table-2 W $\lambda_1 = \lambda_2 = 0$			
	M=0	M=3	M=6	M=10	M=0	M=3	M=6	M=10	M=0	M=3	M=6	M=10
.00	.0114	.0078	.0051	.0028	.0845	.0161	.0072	.0038	.0568	.0153	.0069	.0037
.02	.0093	.0064	.0042	.0023	.0692	.0132	.0059	.0031	.0465	.0125	.0056	.0030
.04	.0076	.0052	.0034	.0019	.0566	.0108	.0048	.0025	.0381	.0102	.0046	.0025
.06	.0062	.0043	.0028	.0015	.0464	.0088	.0039	.0021	.0312	.0084	.0038	.0020
.08	.0051	.0035	.0023	.0012	.0380	.0072	.0032	.0017	.0255	.0069	.0031	.0017
.10	.0042	.0023	.0018	.0010	.0311	.0059	.0026	.0014	.0209	.0056	.0025	.0013
.12	.0034	.0023	.0015	.0008	.0254	.0048	.0022	.0011	.0171	.0046	.0021	.0011
.14	.0028	.0019	.0012	.0007	.0208	.0040	.0018	.0009	.0140	.0038	.0017	.0009

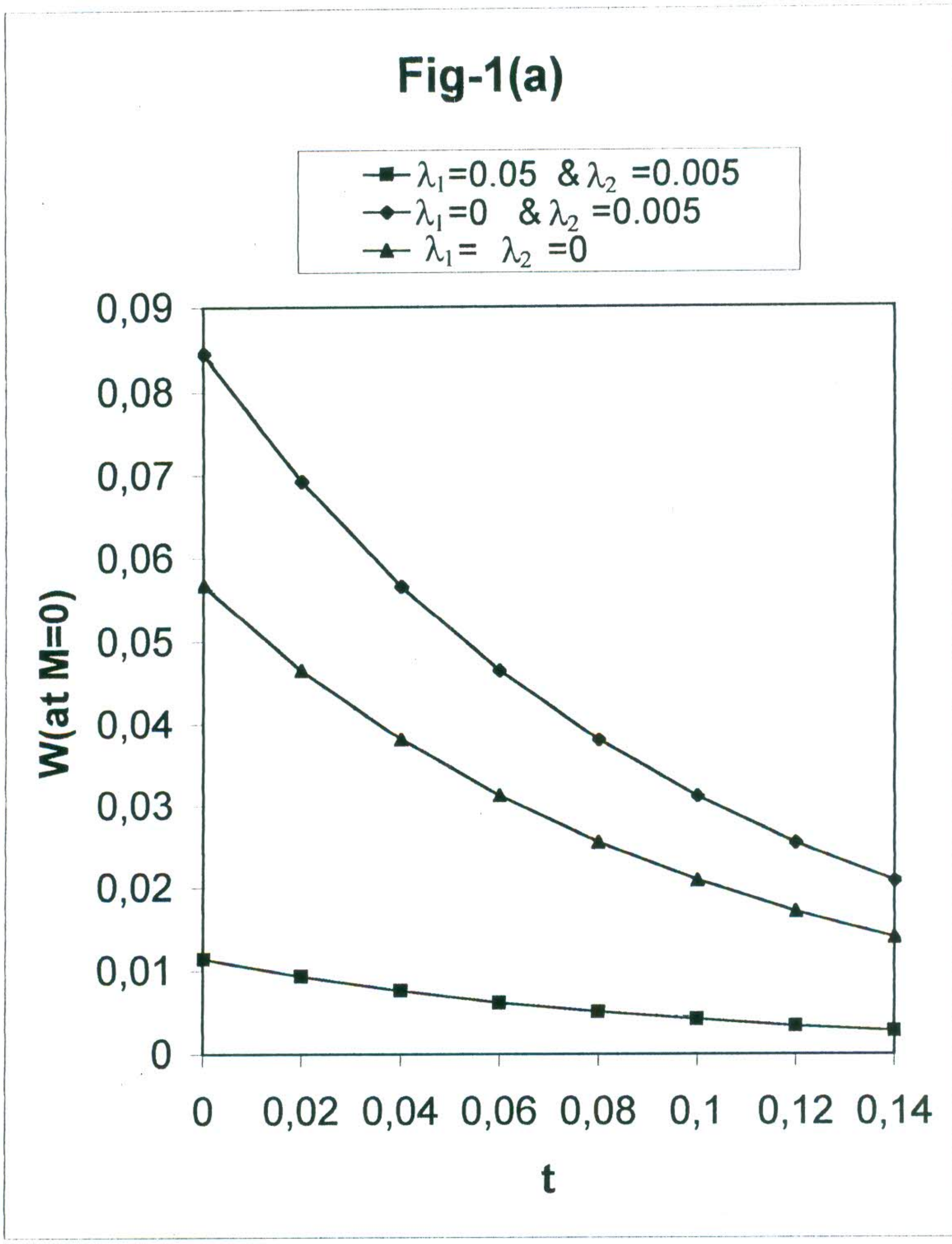

Fig 1(a): Shows the Velocity Profiles at M = 0 for Oldroyd Fluid, Rivilin Ericksen Fluid and Ordinary Viscous Fluid with Time in a Rectangular Channel.

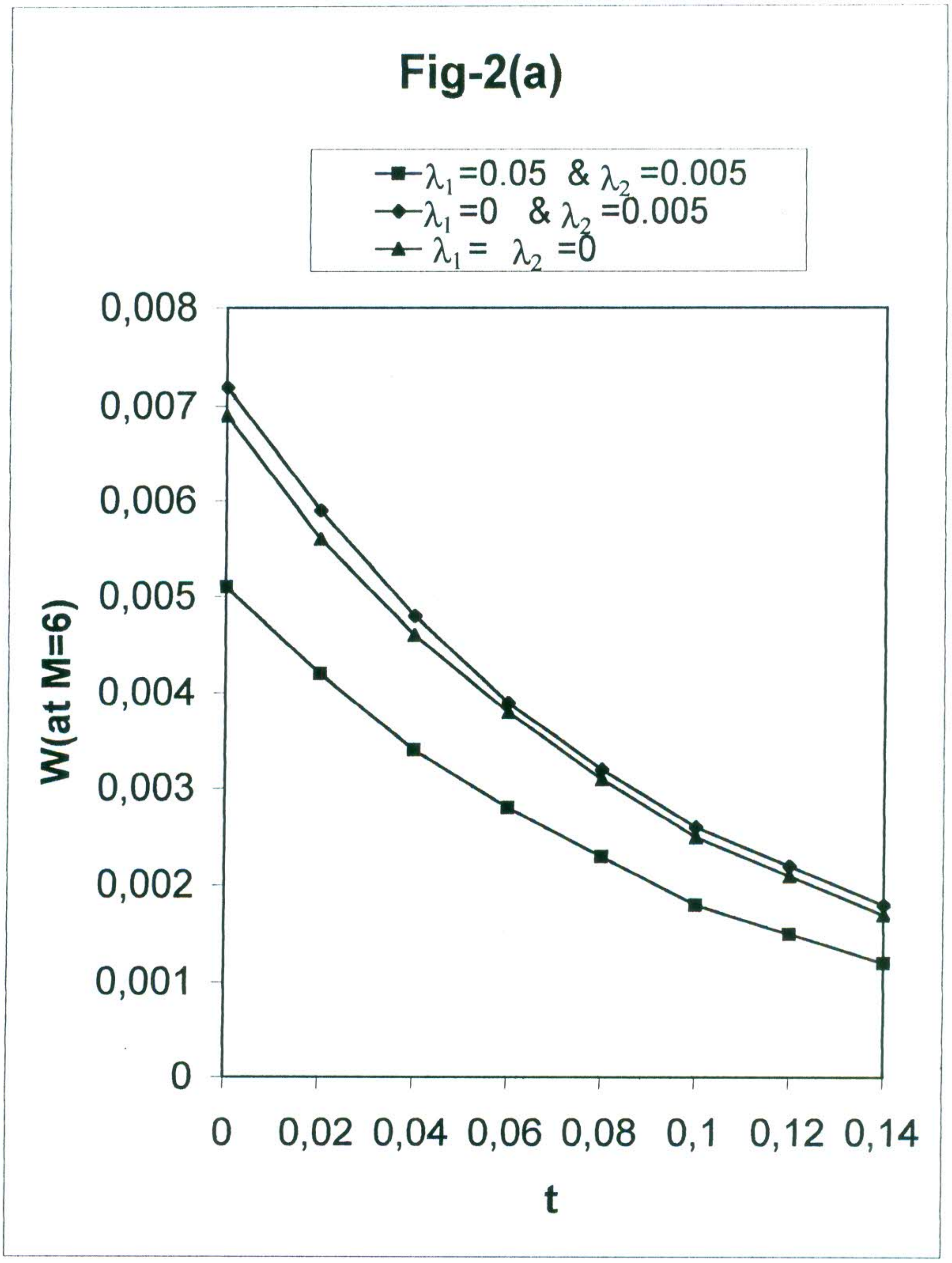

Fig 2(a): Shows the Velocity Profiles at M = 6 for Oldroyd Fluid, Rivilin Ericksen Fluid and Ordinary Viscous Fluid with Time in a Rectangular Channel.

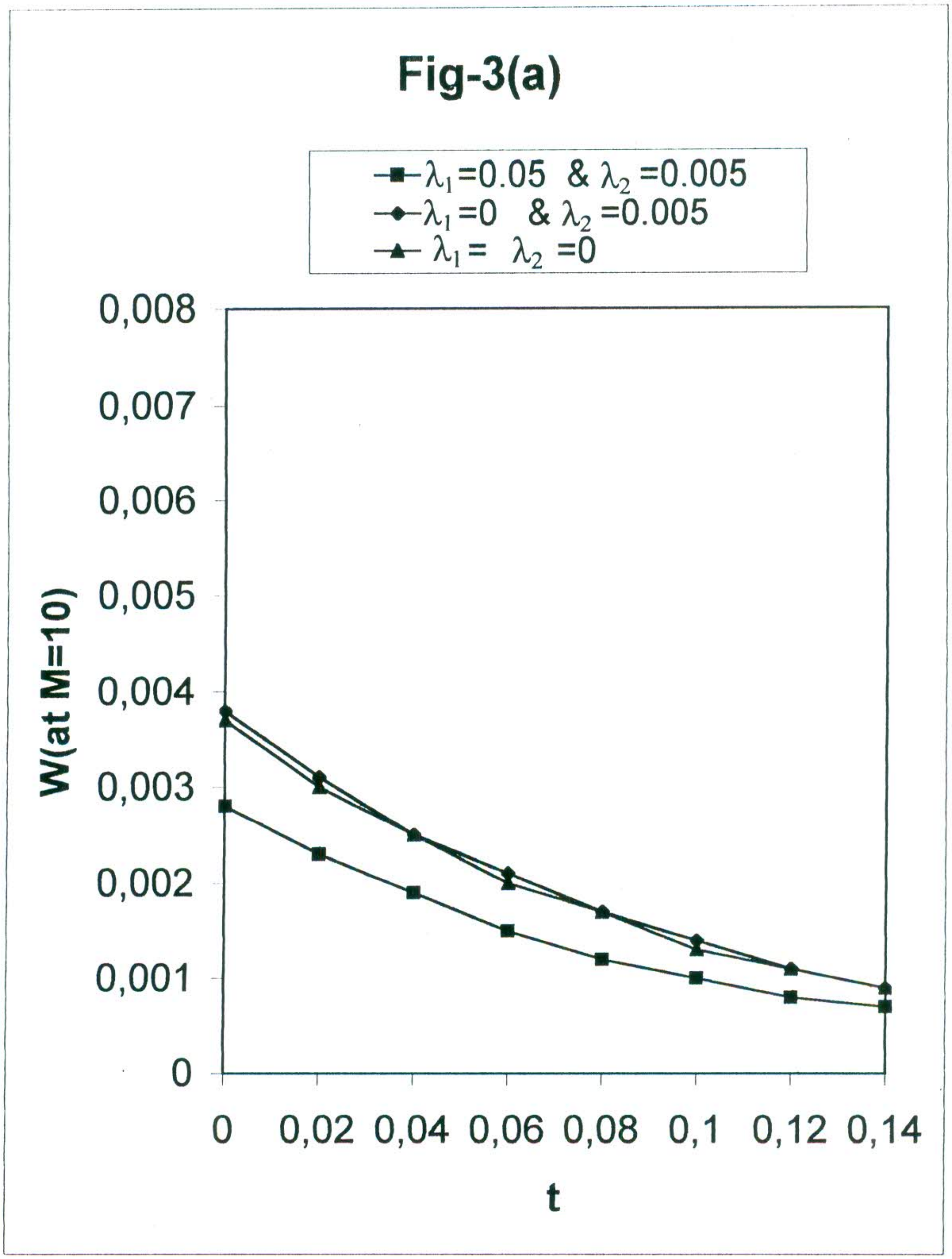

Fig 3(a): Shows the Velocity Profiles at M = 10 for Oldroyd Fluid, Rivilin Ericksen Fluid and Ordinary Viscous Fluid with Time in a Rectangular Channel.

8-7 CONCLUSION

Fig 1(a), 2(a) and 3(a) shows the following results:

1. Velocity of Oldroyd is smaller than Rivlin-Ericksen fluid and ordinary viscous fluid.

2. Velocity of Rivlin-Ericksen fluid as greater than Oldroyd fluid and ordinary viscous fluid.

3. Velocity of ordinary viscous fluid is greater than Oldroyd fluid but smaller than Rivlin-Ericksen fluid.

4. Velocity decrease due to increase of magnetic field in the case of all fluids

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