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Separation Axioms in Intuitionistic Fuzzy Topological Spaces

Ahmed, Estiaq

University of Rajshahi

SEPARATION AXIOMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Ph. D. THESIS

Submitted

By

ESTIAQ AHMED

Department of Mathematics

University of Rajshahi

Rajshahi – 6205, Bangladesh

SEPARATION AXIOMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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BY

ESTIAQ AHMED

Roll No. 11310

Registration No. 526

Session: 2011 – 2012

Department of Mathematics

Faculty of Science

University of Rajshahi

Rajshahi – 6205, Bangladesh

DEDICATED TO MY BROTHER

PROFESSOR DR. ASADUZZAMAN

STATEMENT OF ORIGINALITY

*I declare that the content in my Ph. D. thesis entitled "**SEPARATION AXIOMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**" is original and accurate to the best of my knowledge. I also declare that the materials contained in my research work have not been previously published or written by any person for any degree or diploma.*

September, 2015

(Estiaq Ahmed)

Author

Dr. Md. Sahadat Hossain
Associate Professor
Department of Mathematics
University of Rajshahi
Rajshahi – 6205, Bangladesh

Phone : (off.) +88 - 0721-711108
Cell : +88 - 01914 - 254710
Fax : +88 - 0721-750064
E-mail : sahadat@ru.ac.bd
Web : www.ru.ac.bd /mathematics

Certificate

*This is to certify that the thesis entitled "**SEPARATION AXIOMS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**" submitted by Estiaq Ahmed for the award of the degree of Doctor of Philosophy is a record of the original research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis. This thesis has not been submitted to any other University or Institute for any other degree.*

I wish him a bright future and success in life.

Supervisor

(Dr. Md. Sahadat Hossain)
Associate Professor
Department of Mathematics
University of Rajshahi
Rajshahi – 6205, Bangladesh

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Author

CONTENTS

ABSTRACTS

In 1965, Zadeh[150] introduced to the world the term fuzzy set(FS), as a formalization of vagueness and partial truth, and represents a degree of membership for each member of the universe of discourse to a subset of it. This also provides a natural frame work for generalizing many branches of mathematics such as fuzzy rings, fuzzy vector spaces, fuzzy topology, fuzzy supra topology, fuzzy infra topology. Chang[26] introduced the concepts of fuzzy topological spaces by using fuzzy sets in 1968. Wong [144, 145], Lowen [74], Hutton[60, 61], Khedr[68], Ming and Ming [84, 85] etc. discussed various aspects of fuzzy topology using fuzzy sets. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In the frame work of fuzzifying topology, Shen[118] introduced T_0-space, T_1-space,T_2-space, T_3-space, T_4-space separation axioms in fuzzifying topology. Khedr et. al.[68] introduced and studied the R_0-space, R_1-space separation axioms and found their relations with the T_1-space, T_2-space separation axioms respectively.

After two decades, in 1983, Atanassov[13] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. An intuitionistic fuzzy set (A-IFS), developed by Atanassov[11, 13] is a powerful tool to deal with vagueness. A prominent characteristic of A-IFS is that it assigns to each element a membership degree and a non-

membership degree, and thus, A-IFS constitutes an extension of Zadeh's fuzzy set. He added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1.

In the last few years various concepts in fuzzy sets were extended to intuitionistic fuzzy sets. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [58].

In 1997, Coker [29] introduced the concept of intuitionistic fuzzy topological spaces. Coker et. al.[19, 21, 32, 94] gave some other concepts of intuitionistic fuzzy topological spaces, such as fuzzy continuity, fuzzy compactness, fuzzy connectedness, fuzzy Hausdorff space and separation axioms in intuitionistic fuzzy topological spaces. After this, many concepts in fuzzy topological spaces are being extended to intuitionistic fuzzy topological spaces.

Recently many fuzzy topological concepts such as fuzzy compactness[32], fuzzy connectedness[139], fuzzy separation axioms[20], fuzzy continuity[49], fuzzy g-closed sets[132] and fuzzy g-continuity[134] have been generalized for intuitionistic fuzzy topological spaces.

Demirci[37]presented a Bernays-like axiomatic theory of IFSs involving five primitives and seven axioms. Bustince et. al.[23] defined some intuitionistic fuzzy generators and studied the existence of the equilibrium points and dual points. They presented different characterization theorems of intuitionistic fuzzy generators and a way of constructing IFSs from a fuzzy set and the intuitionistic fuzzy generators. Mondal and Samanta [88] introduced a concept of intuitionistic gradation of openness on fuzzy subsets of a non-empty set and also defined an intuitionistic fuzzy topological space. Deschrijver and Kerre[39], Goguen[47] established the relationships between IFSs, L-fuzzy sets, interval-valued fuzzy sets, and interval-valued IFSs. Bustince and Burillo[24], Deschrijver and Kerre[38] investigated the composition of intuitionistic fuzzy relations. Park[95] defined the notion of intuitionistic fuzzy metric spaces as a natural generalization of fuzzy metric spaces.

Hausdorffness in an intuitionistic fuzzy topological space has been introduced earlier by Coker[29]. Lupianez[76] has also defined new notions of Hausdorffness in the intuitionistic fuzzy sense and obtained some new properties in particular in convergence.

Bayhan and D. Coker[20] introduced fuzzy separation axioms in intuitionistic fuzzy topological spaces. Yue and Fang[149], considered the separation axioms T_0, T_1 and T_2 in an intuitionistic fuzzy (I-fuzzy) topological space. Singh and Srivastava[120, 121] studied separation axioms and also studied α- and α^*-separation axioms in intuitionistic fuzzy topological spaces. Bhattacharjee and Bhaumik[22] has discussed pre-semi separation axioms in intuitionistic fuzzy topological spaces.

The purpose of this thesis is to suggest new definitions of separation axioms in intuitionistic fuzzy topological spaces. We have studied several features of these definitions and the relations among them. We have also shown 'good extension' properties of all these spaces. Our criteria for definitions have been preserved as much as possible the relations between the corresponding separation properties for intuitionistic fuzzy topological spaces.

Our aim is to develop the theories of intuitionistic fuzzy T_0-spaces, intuitionistic fuzzy T_1-spaces, intuitionistic fuzzy T_2-spaces, intuitionistic fuzzy separation axioms. The materials of this thesis have been divided into six chapters. A brief scenario of which we have presented as follows:

In first chapter, chapter one incorporates some of the basic definitions and results of fuzzy sets, intuitionistic sets, intuitionistic fuzzy sets, fuzzy topology, intuitionistic topology, intuitionistic fuzzy topology, fuzzy mappings, intuitionistic fuzzy mappings. These results are ready references for the work in the subsequent chapter. Our work starts from the second chapter.

In second chapter, we have introduced T_0-properties in intuitionistic fuzzy topological spaces and added seven definitions to this list. We have established the relations among them. We have shown that all these definitions satisfy 'good extension' property. It is also shown that these notions are hereditary and productive. We have studied some other properties of these concepts. This chapter is based on the Article [4].

In third chapter, we have studied T_1-properties in intuitionistic fuzzy topological spaces and also adjoined seven definitions to this list. We have established the relations among them. We have shown that all these definitions satisfy 'good extension' property. As earlier we have found that all the definitions are hereditary and productive. Also we have studied some other properties of these concepts. This chapter is based on the Article [5].

In fourth chapter, we have studied T_2-properties in intuitionistic fuzzy topological spaces. Seven definitions are introduced and the relations among them are established. All these definitions satisfy 'good extension' property. These definitions are hereditary and productive. Several other properties of these concepts are also studied. This chapter is based on the Article [6].

In fifth chapter, we have introduced R_0-properties in intuitionistic fuzzy topological spaces. Here we have added seven definitions to this list and established the relations among them. All these definitions satisfy 'good extension' property. We have proved that all the definitions are hereditary. Also we have studied some other properties of these concepts. This chapter is based on the Article [1].

In sixth chapter, we have studied R_1-properties in intuitionistic fuzzy topological spaces. Here we have adjoined seven definitions to this list and established the relations among them. All these definitions satisfy 'good extension' property. We have proved that all these definitions are hereditary and projective. Also we have studied some other properties of these concepts. This chapter is based on the Article [2].

CHAPTER 1

Prerequisites

1.1Introduction:

This chapter incorporates concepts and results of the Fuzzy sets, Intuitionistic sets, Intuitionistic Fuzzy sets, Fuzzy topological spaces, Intuitionistic topological spaces, Intuitionistic Fuzzy topological spaces, subspaces of a Fuzzy topological space, subspace of an Intuitionistic Fuzzy topological space, Fuzzy product topological spaces, Intuitionistic Fuzzy product topological spaces and its properties which are to be used as ready references for understanding the subsequent chapters. Most of the results are quoted from various research papers.

This thesis deals with the study intuitionistic fuzzy topological spaces (IFTS, in short). To present our work in a systematic way in this thesis paper, we consider various concepts and results on fuzzy sets, intuitionistic fuzzy sets, fuzzy topological spaces, intuitionistic fuzzy topological spaces in this chapter.

1.2 Fuzzy set:

Definition 1.2.1[150] Let X be a non-empty set and A be a subset of X. The function $1_A\colon X \longrightarrow [0,\ 1]$ defined by

$$1_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases}$$

is called the characteristic function of A.

Definition 1.2.2[150] A function u from X into the unit interval [0, 1] is called a fuzzy set in X. For every $x \in X$, $u(x) \in [0, 1]$ is called the grade of membership of x in u. Some authors say that u is a fuzzy subset of X in stead of saying that u is a fuzzy set in X.

Definition 1.2.3[84] A fuzzy subset is empty if and only if its grade of membership is identically zero in X. It is denoted by 0.

Definition 1.2.4[84] A fuzzy subset is whole if and only if its grade of membership is identically 1 in X. It is denoted by 1.

Definition 1.2.5[150] Let X be a non-empty set and u and v be two fuzzy subsets of X. Then u is said to be a subset of v if $u(x) \leq v(x)$ for every $x \in X$. It is denoted by $u \subseteq v$.

Definition 1.2.6[150] Let X be a non-empty set and u and v be two fuzzy subsets of X. Then u is said to be equal to v if and only if $u(x) = v(x)$ for every $x \in X$. It is denoted by $u = v$

Definition 1.2.7[150] Let X be a non-empty set and u and v be two fuzzy subsets of X.Then v is said to be the complement of u if $v(x) = 1 - u(x)$, for every $x \in X$. It is denoted by u^C. Obviously $(u^C)^C = u$.

Definition 1.2.8[26] Let X be a non-empty set and u, v be two fuzzy subsets of X. Then the union of u and v is a fuzzy subset of X, written as $u \cup v$ which is defined by $(u \cup v)(x) = \max\{u(x), v(x)\}$, for every $x \in X$.

In general, if Λ be an index set and $A = \{u_i : i \in \Lambda\}$ be a family of fuzzy subsets of X, then the union $\cup u_i$ is defined by $\cup u_i(x) = \sup\{ u_i(x) : i \in \Lambda \}$, $x \in X$.

Definition 1.2.9[26] Let X be a non-empty set and u, v be two fuzzy subsets of X. Then the intersection of u and v is a fuzzy subset of X, written as $(u \cap v)(x) = \min\{u(x), v(x)\}$, for every $x \in X$ and $\cap u_i(x) = \inf\{ u_i(x) : i \in \Lambda \}$, $x \in X$. where u_i are fuzzy subsets of X.

Definition 1.2.10[150] Let X be a non-empty set and u, v be two fuzzy subsets of X. Then the difference of u and v is defined by $u - v = u \cap v^C$.

Definition 1.2.11[85] A fuzzy point in X is a special type of fuzzy set in X with membership function p(x) = r, p(y) = 0, for all $y \neq x$, where $0 < r < 1$. This fuzzy point is said to have support x and value r and this point is denoted by x_r or ${}_r1_x$.

Definition 1.2.12[85] A fuzzy point ρ is said to belong a fuzzy set u in X, that is $\rho \in u$ if and only if $\rho(x) < u(x)$ and $\rho(y) \leq u(y)$, for all $y \neq x$. Evidently, every fuzzy set u can be expressed as the union of all the fuzzy points which belongs to u.

Definition 1.2.13[85] Two fuzzy sets u and v in X are said to be intersected if and only if there exists a point $x \in X$ such that $(u \cap v)(x) \neq 0$. In this case, we say that u and v intersect at x.

Definition 1.2.14[8] A fuzzy singleton 'x_r' is a fuzzy set in X taking value $r \in (0, 1]$ at x and elsewhere. Two fuzzy singletons are said to be distinct if their supports are distinct.

1.3 Intuitionistic set:

Definition 1.3.1[31] An intuitionistic set A is an object having the form $A = (x, A_1, A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A.

Throughout this thesis, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set.

Remark 1.3.2[31] Every subset A on a non-empty set X may obviously be regarded as an intuitionistic set having the form $A' = (A, A^C)$, where $A^C = X \setminus A$ is the complement of A in X.

Definition 1.3.3[31] Let the intuitionistic sets A and B on X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = (A_j^{(1)}, A_j^{(2)})$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(c) $\overline{A} = (A_2, A_1)$, denotes the complement of A.

(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.

(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$.

(f) $\phi_\sim = (\phi, X)$ and $X_\sim = (X, \phi)$.

Definition 1.3.4[31] Let X be a non-empty set and $p \in X$, a fixed element in X. Then the intuitionistic set (IS) $p_\sim = < x, \{p\}, \{p\}^C >$ is called an intuitionistic point (IP, in short) in X.

1.4 Intuitionistic Fuzzy Set:

Definition 1.4.1[13] Let X be a non-empty set and I be the unit interval $[0, 1]$. An intuitionistic fuzzy set A (IFS, in short) in X is an object having form $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$ where $\mu_A: X \to I$ and $\nu_A: X \to I$ denote the degree of membership and the degree of non-membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.

Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1 - \mu_A)$.

Throughout this thesis, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$.

Definition 1.4.2[13] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\nu_A, \mu_A)$.

(4) $A \cap B = (\mu_A \cap \mu_B; \nu_A \cup \nu_B)$.

(5) $A \cup B = (\mu_A \cup \mu_B; \nu_A \cap \nu_B)$.

(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Definition 1.4.3[27] Let $\{ A_i : i \in J \}$ be an arbitrary family of IFSs in X. Then

(a) $\cap A_i = (\cap \mu_{A_i}, \cup \upsilon_{A_i})$.

(b) $\cup A_i = (\cup \mu_{A_i}, \cap \upsilon_{A_i})$.

Definition 1.4.4[33] Let $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP, in short) of X is an IFS of X defined by

$$x_{(\alpha,\ \beta)}(y) = \begin{cases} (\alpha,\ \beta) & \text{if } y = x \\ (0,\ 1) & \text{if } y \neq x \end{cases}$$

In this case, x is called the support of $x_{(\alpha,\ \beta)}$ and α and β are called the value and non-value of $x_{(\alpha,\ \beta)}$, respectively. An IFP $x_{(\alpha,\ \beta)}$ is said to belong to an IFS $A = (\mu_A, \nu_A)$ of X, denoted by $x_{(\alpha,\ \beta)} \in A$, if $\alpha \leq \mu_A(x)$, $\beta \geq \nu_A(x)$.

1.5 Laws of the Algebra of Fuzzy Sets:

Idempotent laws, associative law, commutative law, distributive laws, identity law, Demorgan's laws also hold in the case of fuzzy sets as in ordinary set theory. But the complement laws are not necessarily true.

For example, if $X = \{a, b, c\}$ and u is a fuzzy subset of X which is defined by

$u = \{(a, 0.2), (b, 0.7), (c, 1)\}$,

then $u^C = \{ (a, 0.8), (b, 0.3), (c, 0) \}$.

Now $u \cup u^C = \{(a, 0.8), (b, 0.7), (c, 1)\} \neq 1$,

and $u \cap u^C = \{(a, 0.2), (b, 0.3), (c, 0)\} \neq 0$.

In ordinary sets $u \cap v = \phi$ iff $u \subseteq v^C$. But in fuzzy sets, the reverse is not necessarily true.

For example, if $v = \{(a, 0.6), (b, 0.2), (c, 0)\}$, then $u \subseteq v^C$.

$u \cap v = \{(a, 0.2), (b, 0.2), (c, 0)\} \neq 0$.

Definition 1.5.1[92] For $u \in I^X$ we define

(a) $u_\alpha = \{x : x \in X \text{ and } \alpha \leq u(x)\}$ is the weak α-cut of u, where $\alpha \in (0, 1)$.

The 1-cut is called the kernel of u and it is denoted by ker(u).

(b) $u_\alpha^+ = \{x : x \in X \text{ and } \alpha > u(x)\}$ is the strong α-cut of u, where $\alpha \in (0, 1)$.

The strong 0-cut of u is called the support of u and it is denoted supp(u).

(c) $\text{hgt}(u) = \sup_{x \in X} u(x)$, is the height of u.

1.6 Fuzzy Mappings:

Definition 1.6.1[26] Let f be a mapping from a set X into a set Y and u be a fuzzy subset of X. Then f and u induce a fuzzy subset v of Y defined by

$$f(u)(y) = v(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} u(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 1.6.2[26] Let f be a mapping from a set X into a set Y and v be a fuzzy subset of Y. Then the inverse of v, written as $f^{-1}(v)$ is a fuzzy subset of X and is defined by $(f^{-1}(v))(x) = v(f(x))$, for $x \in X$.

Some Properties of Fuzzy Subsets Induced by Mappings:

Let f be a mapping from X into Y, u be a fuzzy subset of X and v be a fuzzy subset of Y. Then the following properties are true.

(a) $f^{-1}(v^C) = (f^{-1}(v))^C$, for any fuzzy subset v of Y.

(b) $f(u^C) \supseteq (f(u))^C$, for any fuzzy subset u of X.

(c) $v_1 \subseteq v_2 \Longrightarrow f^{-1}(v_1) \subseteq f^{-1}(v_2)$, where v_1 and v_2 are two fuzzy subsets of Y

(d) $u_1 \subseteq u_2 \Longrightarrow f(u_1) \subseteq f(u_2)$, where u_1 and u_2 are two fuzzy subsets of X.

(e) $v \supseteq f(f^{-1}(v)$, for any subset of Y.

(f) $u \subseteq f^{-1}(f(u))$, for any subset u of X.

1.7 Intuitionistic Fuzzy Mappings:

Definition 1.7.1[13] Let X and Y be two nonempty sets and $f : X \to Y$ be a function. If $B = \{(y,\ \mu_B(y),\ \nu_B(y)) \mid y \in Y\}$ is an IFS in Y, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \{(x,\ f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)) \mid x \in X\}$ and the image of A under f, denoted by $f(A) = \{(y,\ f(\mu_A),\ f(\nu_A)) \mid y \in Y\}$is an IFS of Y, where for each $y \in Y$

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ 1 & \text{otherwise.} \end{cases}$$

1.8 Topology:

Definition 1.8.1[73] Let X be a non-empty set and a family T of subsets of X is said to be topology on X if

(1) $X,\ \phi \in T$.

(2) $A,\ B \in T \implies A \cap B \in T$.

(3) $A_i \in T \implies \cup A_i \in T$, for each $i \in \Lambda$.

The pair $(X,\ T)$ is called topological space. Any member $U \in T$ is called open set of the topology T and its complement denoted by U^C is called closed set in the topology T.

1.9 Fuzzy Topology:

Definition 1.9.1[26] Let X be a non-empty set, $I = [0,\ 1]$ be the unit interval, and I^X be the collection of all mappings from X into I, that is the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X satisfying the following conditions.

(1) $0,\ 1 \in t$.

(2) If $u_1,\ u_2 \in t$, then $u_1 \cap u_2 \in t$.

(3) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} \cup\, u_i \in t$.

The pair $(X,\ t)$ is called a fuzzy topological space (fts, in short) and any members of t is called fuzzy open set. A fuzzy set v in t is called fuzzy closed set if $1 - v \in t$.

Example: Let X = { a, b, c, d }, t = {0, 1, u, v}, where

1 = {(a, 1), (b, 1), (c, 1), (d, 1)}.

0 = {(a, 0), (b, 0), (c, 0), (d, 0)}.

u ={(a, 0.2), (b, 0.4), (c, 0.5), (d, 0.8)}.

v ={(a, 0.3), (b, 0.6), (c, 0.8), (d, 0.9)}.

Then (X, t) is a fuzzy topological space.

Definition 1.9.2[85] Let u be a fuzzy set in $(X,\ t)$. The interior of u is defined as the union of all t-open fuzzy sets contained in u. It is denoted by u^0. Evidently u^0 is the largest open fuzzy set contained in u and $(u^0)^0 = u^0$.

Definition 1.9.3[85] The intersection of all the t-closed fuzzy sets containing u is called the closure of u denoted by $\overline{u}$. Obviously $\overline{u}$ is the smallest closed set containing u and $\overline{u} = \overline{\overline{u}}$.

Definition 1.9.4[142] A fuzzy set n in a fts (X, t) is called a neighborhood of a point $x \in X$, if and only if there exists $u \in t$ such that $u \subseteq n$ and $u(x) = n(x) > 0$.

Example: Let us consider the example 1.9.1 and n ={(a, 0.5), (b, 0.6), (c, 0.7), (d, 0.8)}. Hence n is a neighborhood of $d \in X$. Since $u \in t$ such that $u \subseteq n$ and $u(d) = n(d) > 0$. Similarly n_1 ={(a, 0.3), (b, 0.5), (c, 0.6), (d, 0.8)} is a neighborhood of $d \in X$. We denote the family of all neighborhoods of x by N_x.

Definition 1.9.5[84] A fuzzy set u in a fts (X, t) is called a neighborhood of a fuzzy point x_r iff there exists a fuzzy set $u_1 \in t$ such that $x_r \in u_1 \subseteq u$. A neighborhood is called an open neighborhood if u is open. The family consisting of all the neighborhoods of x_r is called the system of neighborhoods of x_r.

1.10 Intuitionistic Topology:

Definition 1.10.1[27] An intuitionistic topology on a set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(1) $\phi_\sim, X_\sim \in \tau$.

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.

In this case, the pair (X, τ) is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS, in short) in X.

1.11 Intuitionistic Fuzzy Topology:

Definition 1.11.1[29] An intuitionistic fuzzy topology (IFT, in short) on X is a family t of IFS's in X which satisfies the following axioms:

(1) $0_\sim, 1_\sim \in t$.

(2) If $A_1, A_2 \in t$, then $A_1 \cap A_2 \in t$.

(3) If $A_i \in t$ for each i, then $\cup A_i \in t$.

The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS, in short). Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS, in short) in X. The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

Definition 1.11.2[29] Let (X, t) be an intuitionistic fuzzy topological space and A an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A is defined by

(i) Cl(A) = $\cap \{F : A \subseteq F, F^c \in t\}$

(ii) int(A) = $\cup \{G : A \supseteq G, G \in t\}$.

Definition 1.11.3[71] Let $P_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point (IFP) in intuitionistic fuzzy topological space $(X,\ t)$. An Intuitionistic fuzzy set A in X is called an intuitionistic fuzzy neighborhood (IFN) of $P_{(\alpha,\ \beta)}$ if there exists an intuitionistic fuzzy open set B in X such that $P_{(\alpha,\ \beta)} \in B \subseteq A$.

1.12 Open, Closed and Continuous Fuzzy Mappings:

Definition 1.12.1[85] The function $f:(X,\ t) \longrightarrow (Y,\ s)$ is called fuzzy continuous if and only if for every $v \in s$ such that $f^{-1}(v) \in t$. The function f is called fuzzy homeomorphism if and only if f is bijective and both f and f^{-1} are fuzzy continuous.

Definition 1.12.2[78] The function $f:(X,\ t) \longrightarrow (Y,\ s)$ is called fuzzy open if and only if for every fuzzy open set u in $(X,\ t)$, $f(u)$ is fuzzy open set in $(Y,\ s)$.

Definition 1.12.3[85] The function $f:(X,\ t) \longrightarrow (Y,\ s)$ is called fuzzy closed if and only if for every fuzzy closed set u in $(X,\ t)$, $f(u)$ is fuzzy closed set in $(Y,\ s)$.

Proposition 1.12.4[85] Let $f:(X,\ t) \longrightarrow (Y,\ s)$ be a fuzzy continuous function, then the following properties hold:

(a) For every s-closed v, $f^{-1}(v)$ is t-closed.

(b) For each fuzzy point ρ in X and each neighborhood v of $f(\rho)$, there exists a neighborhood u of ρ such that $f(u) = v$.

(c) For any fuzzy set u in X, $f(\overline{u}) \subseteq \overline{f(u)}$.

(d) For any fuzzy set v in Y, $\overline{f^{-1}(v)} \subseteq f^{-1}(\overline{v})$.

Proposition 1.12.5[78] Let $f : (X, t) \longrightarrow (Y, s)$ be a fuzzy open function, then the following properties hold:

(a) $f(u^0) \subseteq (f(u))^0$, for each fuzzy set u in X.

(b) $(f^{-1}(v))^0 \subseteq f^{-1}(v^0)$, for each fuzzy set v in Y.

Proposition 1.12.6[78] Let $f : (X, t) \longrightarrow (Y, s)$ be a function. Then f is closed if and only if $\overline{f(u)} \subseteq f(\overline{u})$, for each fuzzy set u in X.

1.13 Open, Closed and Continuous Intuitionistic Mappings:

Definition 1.13.1[28] Let (X, τ) and (Y, ϕ) be two intuitionistic topologies and $f : X \longrightarrow Y$ be a function. Then f is said to be continuous iff the pre-image of each intuitionistic set (IS) in ϕ is an IS in τ.

Definition 1.13.2[28] Let (X, τ) and (Y, ϕ) be two intuitionistic topologies and $f : X \longrightarrow Y$ be a function. Then f is said to be open iff the image of each intuitionistic set (IS) in τ is an IS in ϕ.

1.14 Open, Closed and Continuous Intuitionistic Fuzzy Mappings:

Definition 1.14.1[29] Let (X, t) and (Y, s) be intuitionistic fuzzy topological spaces. Then a map $f : X \longrightarrow Y$ is said to be continuous if

(1) $f^{-1}(B)$ is an intuitionistic fuzzy open set (IFOS) of X for each IFOS B of Y.

(2) $f^{-1}(B)$ is an intuitionistic fuzzy closed set (IFCS) of X for each IFCS B of Y.

(3) Open if f(A) is IFOS of Y for each IFOS A of X.

(4) Closed if f(A) is IFCS of Y for each IFCS A of X.

(5) A homeomorphism if f is bijective, continuous and open.

1.15 Fuzzy Subspace, Base and Subbase:

Definition 1.15.1[84] Let (X, t) be a fuzzy topological space and A be an ordinary subset of X. The class $t_A = \{u|A : u \in t\}$, determines a fuzzy topology on A. This topology is called the subspace fuzzy topology on A.

Definition 1.15.2[145] Let (X, t) be a fuzzy topological space. A subfamily B of t is a base for t if and only if each member of t can be expressed as the union of some members of B.

Definition 1.15.3[145] Let (X, t) be a fuzzy topological space. A subfamily S of t is a sub-base for t if and only if the family of finite intersection of members of S forms a base for t.

1.16 Intuitionistic Base and Subbase:

Definition 1.16.1[28] Let (X, τ) intuitionistic topology on X

(a) A family $\beta \subseteq \tau$ is called a base for (X, τ) iff each member of τ can be written as a union of elements of β.

(b) A family $\gamma \subseteq \tau$ is called a subbase for (X, τ) iff the family of finite intersections of elements in γ forms a base for (X, τ).

1.17 Product Fuzzy Topology:

Definition 1.17.1[145] If u_1 and u_2 are two fuzzy subsets of X and Y, respectively. Then the Cartesian product $u_1 \times u_2$ of two fuzzy subsets u_1 and u_2 is a fuzzy subsets of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min \{(u_1(x), u_2(y)\}$, for each pair $(x, y) \in X \times Y$.

Definition 1.17.2[84] Let $\{X_i, i \in \Lambda\}$ be any class of sets and let X denoted the Cartesian product of these sets, that is, $X = \prod_{i\in\Lambda} X_i$. Note that X consists of all points $p =< a_i, i \in \Lambda >$, where $a_i \in X_i$. Recall that, for each $j_0 \in \Lambda$, we define the projection π_{j_0} from the product set X to the coordinate space X_{j_0}, that is, $\pi_{j_0} : X \longrightarrow X_{j_0}$ by $C(< a_i, i \in \Lambda >) = a_{j_0}$. These projections are used to define the product topology.

Definition 1.17.3[145] If (X_1, t_1) and (X_2, t_2) are two fuzzy topological spaces. If $X = X_1 \times X_2$ is the usual product, t is the coarsest fuzzy topology on X, and each projection $\pi_i : X \longrightarrow X_i$, i = 1, 2, is fuzzy continuous, then the pair (X, t) is called the product space of the fuzzy topological spaces (X_1, t_1) and (X_2, t_2).

Proposition 1.17.4[16] If u is a fuzzy subset of a fuzzy topological space $(X,\ t_1)$ and v is a fuzzy subset of a fuzzy topological space $(Y,\ t_2)$, then $\overline{u \times v} \subseteq \overline{u} \times \overline{v}$.

Definition 1.17.5[144] Let $\{\{X_\alpha\colon \alpha \in \Lambda\}$ be a family of nonempty sets. Let $X = \prod_{\alpha\in\Lambda} X_\alpha$ be the usual product of X_α's and π_α be the projection from X into X_α. Further assume that each X_α is an fts with fuzzy topology t_α. Now the fuzzy topology generated by $\{{\pi_\alpha}^{-1}(b_\alpha) : b_\alpha \in t_\alpha,\ \alpha \in \Lambda\}$ as a sub-base, is called the product fuzzy topology on X. Clearly if w is a basic element in the product, then there exist $\alpha_1,\ \alpha_2,\ \alpha_3,\ \ldots\ldots\ldots\ldots\ldots\ \alpha_n \in \Lambda$ such that $w(x) = \min\{b_\alpha(x_\alpha) : \alpha = 1, 2, 3, \ldots\ldots\ldots\ldots n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$.

1.18 Product Intuitionistic Fuzzy Topology:

Definition 1.18.1[21] Let $A = (X, \mu_A, \upsilon_A)$ and $B = (Y, \mu_B, \upsilon_B)$ be IFSs of X and Y respectively. Then the product of intuitionistic fuzzy sets A and B denoted by $A \times B$ is defined by $A \times B = \{X \times Y,\ \mu_A \times \mu_B,\ \ \upsilon_A \times \upsilon_B\}$ where $(\mu_A \times \mu_B)(x,\ y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\upsilon_A \times \upsilon_B)(x,\ y) = \max\{\upsilon_A(x), \upsilon_B(y)\}$ for all $(x,\ y) \in X \times Y$. Obviously, $0 \le \mu_A \times \mu_B + \upsilon_A \times \upsilon_B \le 1$. This definition can be extended to an arbitrary family of IFSs as follows:

If $A_i = \{(\mu_{A_i}, \upsilon_{A_i}), i \in J\}$ is a family of IFSs in X_i, then their product is defined as the IFS in $\prod X_i$ given by $\prod A_i = (\prod \mu_{A_i},\ \prod \upsilon_{A_i})$ where $\prod \mu_{A_i}(x) = \inf \mu_{A_i}(x_i)$, for all $x = \prod x_i \in X$ and $\prod \upsilon_{A_i}(x) = \sup \upsilon_{A_i}(x_i)$, for all $x = \prod x_i \in X$.

Definition 1.18.2[21] Let $(X_i,\ t_i)$, $i = 1,\ 2$ be two IFTSs, then the product $t_1 \times t_2$ on $X_1 \times X_2$ is defined as the IFT generated by $\{\rho_i^{-1}(U_i) : U_i \in t_i,$ $i = 1,\ 2\}$, where $\rho_i : X_1 \times X_2 \to X_i$, $i = 1,\ 2$ are the projection maps and the IFTS $(X_1 \times X_2,\ t_1 \times t_2)$ is called product IFTS.

Definition 1.18.3[52] Let $f_1 : X_1 \longrightarrow Y_1$ and $f_2 : X_2 \longrightarrow Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is defined by

$(f_1 \times f_2)\ (x_1 \times x_2) = \{\ f_1(x_1), f_2(x_2)\ ),\ \forall\ (x_1 \times x_2) \in X_1 \times X_2\}$.

Definition 1.18.4[52] Let $f : X \longrightarrow Y$ be a function. The graph $g : X \longrightarrow X \times Y$ of f is defined by $g(x) = (\ x,\ f(x)),\ \forall\ x \in X$.

Definition 1.18.5[52] Let $f_i : X_i \longrightarrow Y_i$ (i = 1, 2) be functions and U, V intuitionistic fuzzy sets of Y_1 and Y_2, respectively, then $(f_1 \times f_2)^{-1}$ $(U \times V) =$ $f_1^{-1}(U) \times f_2^{-1}(V)$.

1.19 Good Extensions in Fuzzy Topological Spaces:

Definition 1.19.1[104] Let f be a real valued function on a topological space. If $\{x : f(x) > \alpha\}$ is open for every real α, then f is called lower semi continuous function.

Definition 1.19.2 [74] Let X be a nonempty set and T be a topology on X. Let $t = \omega(T)$ be the set of all lower semi continuous (lsc) functions from $(X,\ T)$ to

I (with usual topology). Thus $\omega(T) = \{u \in I^X : u^{-1}(\alpha, 1] \in T\}$, for each $\alpha \in [0, 1)$ is a fuzzy topology on X.

Let P be the property of a topological space (X, T) and FP be its fuzzy topological analogue. Then FP is called a 'good extension' of P "iff the statement (X, T) has P iff (X, $\omega(T)$) has FP" holds good extension for every topological space (X, T).

Definition 1.19.3[74] Let (X, T) be an ordinary topological space. The set of all lower semi continuous functions from (X, T) into the closed unit interval equipped with the usual topology constitutes a fuzzy topology associated with (X, T) and is denoted by (X, $\omega(T)$).

1.20 Good Extensions in intuitionistic Fuzzy Topological Spaces:

Theorem 1.20.1[4](theorem 2.1.1) Let (X, τ) be an intuitionistic topological space and let $t = \{1_A : A \in \tau\}$, $1_{(A_1, A_2)} = (1_{A_1}, 1_{A_2})$, then (X, t) is the corresponding intuitionistic fuzzy topological space of (X, τ).

1.21 Separation axioms in fuzzy topological spaces:

Definition 1.21.1[9] A fuzzy topological space (X, t) is called

(a) F-T_0 if $\forall$ x, y $\in$ X, x $\neq$ y, $\exists$ u $\in$ t such that $u(x) \neq u(y)$.

(b) F-T_1 if $\forall$ x, y $\in$ X, x $\neq$ y, $\exists$ u, v $\in$ t such that $u(x) = 1$, $u(y) = 0$ and $v(x) = 1$, $v(y) = 1$.

Definition 1.21.2[53] Let $(X,\ t)$ be a fuzzy topological space and $\alpha \in [0,1)$, then

(a) $(X,\ t)$ is T_0(i) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u \in t$ such that $u(x) = 1,\ u(y) = 0$ or $\exists\ v \in t$ such that $v(x) = 0,\ v(y) = 1$.

(b) $(X,\ t)$ is α-T_0(ii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u \in t$ such that $u(x) = 1,\ u(y) \leq \alpha$ or $\exists\ v \in t$ such that $v(x) \leq \alpha,\ v(y) = 1$.

(c) $(X,\ t)$ is α-T_0(iii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u \in t$ such that $u(x) = 0,\ u(y) > \alpha$ or $\exists\ v \in t$ such that $v(x) > \alpha,\ v(y) = 0$.

(d) $(X,\ t)$ is α-T_0(iv) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u \in t$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ or $\exists\ v \in t$ such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$.

(e) $(X,\ t)$ is α-T_0(v) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u \in t$ such that $u(x) \neq u(y) = 1$.

Definition 1.21.3[54] Let $(X,\ t)$ be a fuzzy topological space and $\alpha \in [0,1)$, then

(a) $(X,\ t)$ is T_1(i) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u, v \in t$ such that $u(x) = 1,\ u(y) = 0$ and $v(x) = 0,\ v(y) = 1$.

(b) $(X,\ t)$ is α-T_1(ii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u, v \in t$ such that $u(x) = 1,\ u(y) \leq \alpha$ and $v(x) \leq \alpha,\ v(y) = 1$.

(c) $(X,\ t)$ is α-T_1(iii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u, v \in t$ such that $u(x) = 0,\ u(y) > \alpha$ and $v(x) > \alpha,\ v(y) = 0$.

(d) $(X,\ t)$ is α-T_1(iv) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$, $\exists\ u, v \in t$ such that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ and $0 \leq v(x) \leq \alpha < v(y) \leq 1$.

(e) $(X,\ t)$ is α-$T_1(v)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) < u(y)$ and $v(x) > v(y)$.

Definition 1.21.4[55] Let $(X,\ t)$ be a fuzzy topological space and $\alpha \in [0,1)$, then

(a) $(X,\ t)$ is α-$T_2(i)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) = 1 = v(y) = 1$ and $u \cap v = 0$.

(b) $(X,\ t)$ is α-$T_1(ii)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) = 1 = v(y) = 1$ and $u \cap v \leq \alpha$.

(c) $(X,\ t)$ is α-$T_1(iii)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) > \alpha$ and $v(y) > \alpha$ and $u \cap v = 0$.

(d) $(X,\ t)$ is α-$T_1(iv)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) > \alpha$ and $v(y) > \alpha$ and $u \cap v \leq \alpha$.

(e) $(X,\ t)$ is α-$T_1(v)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$, $\exists\ u,\ v \in t$ such that $u(x) > 0$ and $v(y) > 0$ and $u \cap v = 0$.

Definition 1.21.5[56] Let $(X,\ t)$ be a fuzzy topological space and $\alpha \in [0,1)$, then

(a) $(X,\ t)$ is $R_0(i)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$ whenever $\exists\ u \in t$ with $u(x) = 1,\ u(y) = 0$, then $\exists\ v \in t$ with $v(x) = 0,\ v(y) = 1$.

(b) $(X,\ t)$ is α-$R_0(ii)$ space $\Leftrightarrow \forall\ x,\ y \in X$ with $x \neq y$ whenever $\exists\ u \in t$ with $u(x) = 1,\ u(y) = 0$, then $\exists\ v \in t$ with $v(x) = 0,\ v(y) = 1$.

(c) (X, t) is α-R_0(iii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ u \in t$ with $u(x) = 0$, $u(y) > \alpha$, then $\exists\ v \in t$ with $v(x) > \alpha$, $v(y) = 0$.

(d) (X, t) is α-R_0(iv) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ u \in t$ with $0 \leq u(x) \leq \alpha < u(y) \leq 1$, then $\exists\ v \in t$ with $0 \leq v(y) \leq \alpha < v(x) \leq 1$.

(e) (X, t) is α-R_0(v) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ u \in t$ with $u(x) < u(y)$, then $\exists\ v \in t$ with $v(x) > v(y)$.

Definition 1.21.6[56] Let (X, t) be a fuzzy topological space and $\alpha \in [0, 1)$, then

(a) (X, t) is R_1(i) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ w \in t$ with $w(x) \neq w(y)$, then $\exists\ u, v \in t$ such that $u(x) = 1 = v(y)$ and $u \cap v = 0$.

(b) (X, t) is α-R_1(ii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ w \in t$ with $w(x) \neq w(y)$, then $\exists\ u, v \in t$ such that $u(x) = 1 = v(y)$ and $u \cap v \leq \alpha$.

(c) (X, t) is α-R_1(iii) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ w \in t$ with $w(x) \neq w(y)$, then $\exists\ u, v \in t$ such that $u(x) > \alpha$, $v(y) > \alpha$ and $u \cap v = 0$.

(d) (X, t) is α-R_1(iv) space $\Leftrightarrow \forall\ x, y \in X$ with $x \neq y$ whenever $\exists\ w \in t$ with $w(x) \neq w(y)$, then $\exists\ u, v \in t$ such that $u(x) > \alpha$, $v(y) > \alpha$ and $u \cap v \leq \alpha$.

CHAPTER 2

On Intuitionistic Fuzzy T_0-Spaces

The concepts of T_0-space are established by Ali [9], Hossain[53] in fuzzy topological spaces. Shen[118] introduced T_0-space in fuzzifying topology. Li [72] studied T_0-space in L-fuzzy topological spaces. Bayhan and Coker [19] introduced T_0-space in intuitionistic fuzzy topological spaces. Yue and Fang [149] considered the separation axioms T_0-space in an intuitionistic fuzzy (I-fuzzy) topological space.

In this chapter, we mention seven possible notions of intuitionistic fuzzy T_0 (in short, IF-T_0) space. Firstly, we establish the relations among them. We show that all these notions satisfy 'good extension' property. Furthermore, it proves that these intuitionistic fuzzy T_0-spaces are hereditary and productive. Finally, we observe that all concepts are preserved under one-one, onto and continuous mappings.

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2.1 Definition and Properties:

Theorem 2.1.1 Let (X, τ) be an intuitionistic topological space and let $t = \{1_A : A \in \tau\}$, $1_{(A_1, A_2)} = (1_{A_1}, 1_{A_2})$, then (X, t) is an intuitionistic fuzzy topological space.

Proof: (i) $\phi_\sim = (\phi, X) \in \tau \implies 1_{\phi_\sim} = (1_\phi, 1_X) = (0^\sim, 1^\sim) = 0_\sim \in t$. Hence $\phi \in \tau \Leftrightarrow 0_\sim \in t$. Now $X_\sim = (X, \phi) \in \tau \implies 1_{X_\sim} = (1_X, 1_\phi) = (1^\sim, 0^\sim) = 1_\sim \in t$. Hence $X_\sim \in \tau \Leftrightarrow 1_\sim \in t$.

(ii) Let $G_1, G_2 \in \tau$, then $G_i = (G_i^1, G_i^2) \in \tau$ (i = 1, 2). Now $1_{G_i} = (1_{G_i^1}, 1_{G_i^2})$. Hence $G_i \in \tau \Leftrightarrow 1_{G_i} \in t$ (i = 1, 2). Now $G_1 \cap G_2 \in \tau \Leftrightarrow (G_1^1 \cap G_2^1, G_1^2 \cup G_2^2) \in \tau$. And $1_{G_1 \cap G_2} = (1_{G_1^1 \cap G_2^1}, 1_{G_1^2 \cup G_2^2}) = 1_{G_1} \cap 1_{G_2}$. Hence $G_1 \cap G_2 \in \tau \Leftrightarrow 1_{G_1 \cap G_2} \in t$.

(iii) Let $G_i \in \tau \Leftrightarrow \cup_i G_i = (\cup_i G_i^1, \cap_i G_i^2) \in \tau$ (i = 1,2,3…,). And $1_{\cup_i G_i} = (1_{\cup_i G_i^1}, 1_{\cap_i G_i^2}) = \cup_i 1_{G_i}$. Hence $\cup_i G_i \in \tau \Leftrightarrow 1_{\cup_i G_i} \in t$. Therefore, (X, t) is an intuitionistic fuzzy topological space.

Definition 2.1.2 As defined in theorem 2.1.1 (X, t) is called the intuitionistic fuzzy topological space to the corresponding intuitionistic topological space (X, τ).

Theorem 2.1.3 Every intuitionistic topological space corresponds to an intuitionistic fuzzy topological space but the converse is not true in general.

Proof: First part has been proved in theorem 2.1.1

Conversely, let $X = \{x, y, z\}$ and let t be the intuitionistic fuzzy topology on X where $t = \{ 1_\sim, 0_\sim, (\mu_1, \mu_1^c), (\mu_2, \mu_2^c), (\mu_3, \mu_3^c) \}$ and

$\mu_1(x) = 1,\quad \mu_2(x) = 0.9,\quad \mu_3(x) = 0.8$

$\mu_1(y) = 0.9,\quad \mu_2(y) = 0.8,\quad \mu_3(y) = 0.7$

$\mu_1(z) = 0.8,\quad \mu_2(z) = 0.7,\quad \mu_3(z) = 0.6$

Then (X, t) is an intuitionistic fuzzy topological space but there is no other intuitionistic topological space which corresponds to (X, t).

Definition 2.1.4 An intuitionistic fuzzy topological space (X, t) is called

(1) IF-T_0(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$.

(2) IF-T_0(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) > 0$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) > 0$.

(3) IF-T_0(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$.

(4) IF-T_0(iv) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) > 0$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) > 0$.

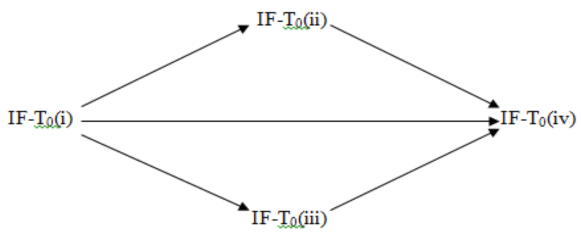
Definition 2.1.5 Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-T_0(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

(b) α-IF-T_0(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ or $\mu_A(y) \geq \alpha$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

(c) α-IF-T_0(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

Theorem 2.1.6 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is IF-T_0(i). We shall prove that (X, t) is IF-T_0(ii). Since (X, t) is IF-T_0(i), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$ $\Longrightarrow$ $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) > 0$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) > 0$. Which is IF-T_0(ii). Hence IF-T_0(i) $\Longrightarrow$ IF-T_0(ii).

Again, suppose (X, t) is IF-T_0(i). We shall prove that (X, t) is IF-T_0(iii). Since (X, t) is IF-T_0(i), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$ $\Longrightarrow$ $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$,

$\nu_A(y) = 1$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$. Which is IF-T_0(iii). Hence IF-T_0(i) $\Longrightarrow$ IF-T_0(iii).

Furthermore, it can prove that IF-T_0(i) $\Longrightarrow$ IF-T_0(iv), IF-T_0(ii) $\Longrightarrow$ IF-T_0(iv) and IF-T_0(iii) $\Longrightarrow$ IF-T_0(iv).

None of the reverse implications is true in general as can be seen from the following examples.

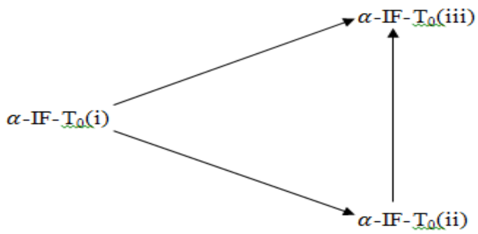
Example (a) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where $A = \{(x, 1, 0), (y, 0, 0.3)\}$. We see that the IFTS (X, t) is IF-T_0(ii) but not IF-T_0(i).

Example (b) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where $A = \{(x, 0.4, 0), (y, 0, 1)\}$. We see that the IFTS (X, t) is IF-T_0(iii) but not IF-T_0(i).

Example (c) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where $A = \{(x, 1, 0), (y, 0, 0.5)\}$. We see that the IFTS (X, t) is IF-T_0(ii) but not IF-T_0(iii).

Example (d) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where $A = \{(x, 0.6, 0), (y, 0, 1)\}$. We see that the IFTS (X, t) is IF-T_0(iii) but not IF-T_0(ii).

Theorem 2.1.7 Let $(X,\ t)$ be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let $\alpha \in (0,\ 1)$. Suppose $(X,\ t)$ is α-IF-T_0(i). We shall prove that $(X,\ t)$ is α-IF-T_0(ii). Since $(X,\ t)$ is α-IF-T_0(i) space, then for all $x,\ y \in X,\ \ x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1,\ \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) \geq \alpha$ or $\mu_A(y) = 1, \nu_A(y) = 0;\ \mu_A(x) = 0, \nu_A(x) \geq \alpha \implies \mu_A(x) \geq \alpha,\ \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) \geq \alpha$ or $\mu_A(y) \geq \alpha,\ \nu_A(y) = 0;\ \mu_A(x) = 0,\ \nu_A(x) \geq \alpha$ for any $\alpha \in (0,\ 1)$. Which α-IF-T_0(ii). Hence α-IF-T_0(i) $\implies$ α-IF-T_0(ii).

Again, let $\alpha \in (0,\ 1)$. Suppose $(X,\ t)$ is α-IF-T_0(ii). We shall prove that $(X,\ t)$ is α-IF-T_0(iii). Since $(X,\ t)$ is α-IF-T_0(ii) space, then for all $x,\ y \in X,\ x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) \geq \alpha,\ \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) \geq \alpha$ or $\mu_A(y) \geq \alpha,\ \nu_A(y) = 0;\ \mu_A(x) = 0,\ \nu_A(x) \geq \alpha \implies \mu_A(x) > 0,\ \nu_A(x) = 0; \mu_A(y) = 0,\ \nu_A(y) \geq \alpha$ or $\mu_A(y) > 0, \nu_A(y) = 0;\ \mu_A(x) = 0,\ \nu_A(x) \geq \alpha$ for any $\alpha \in (0,\ 1)$. Which is α-IF-T_0(iii). Hence α-IF-T_0(ii) $\implies$ α-IF-T_0(iii).

Furthermore, it can prove that α-IF-T_0(i) $\implies$ α-IF-T_0(iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by$\{A\}$, where A = $\{(x, 0.5, 0), (y, 0, 0.5)\}$. For $\alpha = 0.3$, we see that the IFTS (X, t) is α-IF-T_0(ii) but not α-IF-T_0(i).

Example (b) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where A = $\{(x, 0.3, 0), (y, 0, 0.7)\}$. For $\alpha = 0.5$, we see that the IFTS (X, t) is α-IF-T_0(iii) but neither α-IF-T_0(i) nor α-IF-T_0(ii).

Theorem 2.1.8 Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then

(1) β-IF-T_0(i) $\Rightarrow$ α-IF-T_0(i).

(2) β-IF-T_0(ii) $\Rightarrow$ α-IF-T_0(ii).

(3) β-IF-T_0(iii) $\Rightarrow$ α-IF-T_0(iii).

Proof (1): Suppose the IFTS (X, t) is β-IF-T_0(i). We shall prove that (X, t) is α-IF-T_0(i). Since (X, t) is β-IF-T_0(i), then for all $x, y \in X$, $x \neq y$ with $\beta \in (0, 1)$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \beta$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \beta \Rightarrow$ $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \alpha$ as $0 < \alpha \leq \beta < 1$. Which is α-IF-T_0(i). Hence β-IF-T_0(i) $\Rightarrow$ α-IF-T_0(i).

The proofs that β-IF-T_0(ii) $\Rightarrow$ α-IF-T_0(ii) and β-IF-T_0(iii) $\Rightarrow$ α-IF-T_0(iii) are similar.

None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by {A} where $A = \{(x, 1, 0), (y, 0, 0.5)\}$. For $\alpha = 0.3$ and $\beta = 0.8$, we see that the IFTS (X, t) is α-IF-T_0(i) but not β-IF-T_0(i).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by {A} where $A = \{(x, 0.4, 0), (y, 0, 0.4)\}$. For $\alpha = 0.3$ and $\beta = 0.5$, we see that the IFTS (X, t) is α-IF-T_0(ii) but not β-IF-T_0(ii).

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by {A} where $A = \{(x, 0.2, 0), (y, 0, 0.6)\}$. For $\alpha = 0.5$ and $\beta = 0.7$, we see that the IFTS (X, t) is α-IF-T_0(iii) but not β-IF-T_0(iii).

2.2 Subspaces:

Theorem 2.2.1 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$, then

(1) (X, t) is IF-T_0(i) $\Rightarrow$ (U, t_U) is IF-T_0(i).

(2) (X, t) is IF-T_0(ii) $\Rightarrow$ (U, t_U) is IF-T_0(ii).

(3) (X, t) is IF-T_0(iii) $\Rightarrow$ (U, t_U) is IF-T_0(iii).

(4) (X, t) is IF-T_0(iv) $\Rightarrow$ (U, t_U) is IF-T_0(iv).

Proof (1): Suppose $(X,\ t)$ is IF-T_0(i). We shall prove that $(U,\ t_U)$ is IF-T_0(i). Let $x,\ y\ \in U$ with $x \neq y$, then $x,\ y\ \in X$ with $x \neq y$ as $U \subseteq X$. Since $(X,\ t)$ is IF-T_0(i), then there exists $B = (\mu_B\ , \nu_B) \in t$ such that $\mu_B(x) = 1, \nu_B(x) = 0$; $\mu_B(y) = 0,\ \nu_B(y) = 1$ or $\mu_B(y) = 1,\ \nu_B(y) = 0$; $\mu_B(x) = 0,\ \nu_B(x) = 1 \implies (\mu_B|U)(x) = 1,\ (\nu_B|U)(x) = 0;\ (\mu_B|U)(y) = 0,\ (\nu_B|U)(y) = 1$ or $(\mu_B|U)(y) = 1,\ (\nu_B|U)(y) = 0;\ (\mu_B|U)(x) = 0,\ (\nu_B|U)(x) = 1$. Hence $(\mu_B|U,\ \nu_B|U\ ) \in t_U \implies B|U \in t_U$. Therefore, the IFTS $(U,\ t_U)$ is IF-T_0(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 2.2.2 Let $(X,\ t)$ be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{A|U : A \in t\}$ and let $\alpha \in (0,\ 1)$, then

(a) $(X,\ t)$ is α-IF-T_0(i) $\implies$ $(U,\ t_U)$ is α-IF-T_0(i).

(b) $(X,\ t)$ is α-IF-T_0(ii) $\implies$ $(U,\ t_U)$ is α-IF-T_0(ii).

(c) $(X,\ t)$ is α-IF-T_0(iii) $\implies$ $(U,\ t_U)$ is α-IF-T_0(iii).

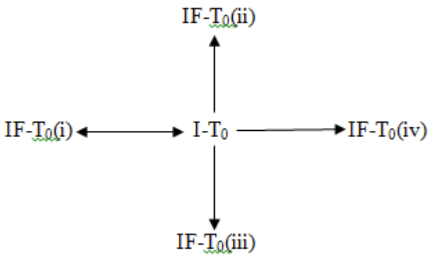
Proof (a): Suppose $(X,\ t)$ is α-IF-T_0(i). We shall prove that $(U,\ t_U)$ is α-IF-T_0(i). Let $x,\ y\ \in U$ with $x \neq y$, then $x,\ y\ \in X$ with $x \neq y$ as $U \subseteq X$. Since $(X,\ t)$ is α-IF-T_0(i), then there exists $B = (\mu_B\ , \nu_B) \in t$ such that $\mu_B(x) = 1,\ \nu_B(x) = 0;\ \mu_B(y) = 0,\ \nu_B(y) \geq \alpha$ or $\mu_B(y) = 1,\ \nu_B(y) = 0$; $\mu_B(x) = 0,\ \nu_B(x) \geq \alpha \implies (\mu_B|U)(x) = 1, (\nu_B|U)(x) = 0;\ (\mu_B|U)(y) = 0,$ $(\nu_B|U)(y) \geq \alpha$ or $(\mu_B|U)(y) = 1,\ (\nu_B|U)(y) = 0;\ (\mu_B|U)(x) = 0$, $(\nu_B|U)(x) \geq \alpha$. Hence $(\mu_B|U,\ \nu_B|U\ ) \in t_U \implies B|U \in t_U$. Therefore, the IFTS $(U,\ t_U)$ is α-IF-T_0(i).

(b) and (c) can be proved in the similar way.

2.3 Good extension:

Definition 2.3.1 An intuitionistic topological space (X, τ) is called intuitionistic T_0-space (I-T_0 space) if for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2) \in \tau$ such that $(x \in C_1$ and $y \in C_2)$ or $(y \in C_1$ and $x \in C_2)$.

Theorem 2.3.2 Let (X, τ) be an intuitionistic topological space and let (X, t) be the intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is I-T_0 space. We shall prove that (X, t) is IF-T_0(i). Since (X, τ) is I-T_0, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2) \in \tau$ such that $(x \in C_1$ and $y \in C_2)$ or $(y \in C_1$ and $x \in C_2)$. Suppose $x \in C_1$ and $y \in C_2 \Longrightarrow 1_{C_1}(x) = 1$, $1_{C_2}(y) = 1$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, then $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) = 1$. Hence $(\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) = 1 \Longrightarrow (X, t)$ is IF-T_0(i). Therefore I-$T_0 \Longrightarrow$ IF-T_0(i).

Conversely, suppose (X, t) is IF-T_0(i). We shall prove that (X, τ) is I-T_0. Since (X, t) is IF-T_0(i), then for all x, y $\in$ X, x $\neq$ y there exists $(1_{C_1}, 1_{C_2}) \in t$ such that $\{1_{C_1}(x) = 1, 1_{C_2}(x) = 0;\ 1_{C_1}(y) = 0,\ 1_{C_2}(y) = 1\}$ or $\{1_{C_1}(y) = 1,$ $1_{C_2}(y) = 0;\ \ 1_{C_1}(x) = 0,\ 1_{C_2}(x) = 1\}$. Suppose $1_{C_1}(x) = 1,\ 1_{C_2}(x) = 0;$ $1_{C_1}(y) = 0,\ \ 1_{C_2}(y) = 1\ \ \Rightarrow\ \ x \in C_1,\ \ x \notin C_2;\ \ y \notin C_1,\ \ y \in C_2$. Hence $(C_1, C_2) \in \tau \Rightarrow$ (X, τ) is I-T_0. Therefore I-T_0 $\Leftrightarrow$ IF-T_0(i).

Furthermore, it can prove that I-T_0 $\Rightarrow$ IF-T_0(ii), I-T_0 $\Rightarrow$ IF-T_0(iii) and I-T_0 $\Rightarrow$ IF-T_0(iv).

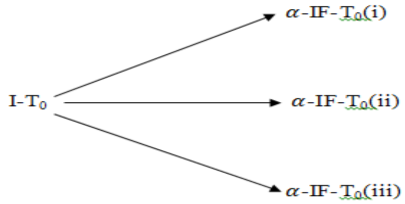
None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let X $= \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A} where A = {(x, 1, 0),(y, 0, 0.3)}. We see that the IFTS (X, t) is IF-T_0(ii) but not I-T_0.

Example (b) Let X $= \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A} where A = {(x, 0.4, 0), (y, 0, 1)}. We see that the IFTS (X, t) is IF-T_0(iii) but not I-T_0.

Example (c) Let X $= \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A} where A = {(x, 0.5, 0), (y, 0, 0.7)}. We see that the IFTS (X, t) is IF-T_0(iv) but not I-T_0.

Theorem 2.3.3 Let (X, τ) be an intuitionistic topological space and let (X, t) be the intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let $\alpha \in (0, 1)$. Suppose (X, τ) is I-T_0. We shall prove that (X, t) is α-IF-T_0(i). Since (X, τ) is I-T_0, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2) \in \tau$ such that $(x \in C_1$ and $y \in C_2)$ or $(y \in C_1$ and $x \in C_2)$. Suppose $x \in C_1$ and $y \in C_2 \Rightarrow 1_{C_1}(x) = 1$, $1_{C_2}(y) = 1$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, then $\mu_A(x) = 1, \upsilon_A(x) = 0$; $\mu_A(y) = 0, \upsilon_A(y) = 1 \Rightarrow \mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) \geq \alpha$ for any $\alpha \in (0, 1)$. Hence $(\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) \geq \alpha \Rightarrow (X, t)$ is α-IF-T_0(i). Therefore I-$T_0 \Rightarrow$ α-IF-T_0(i).

Furthermore, it can prove that I-$T_0 \Rightarrow$ α-IF-T_0(ii), I-$T_0 \Rightarrow$ α-IF-T_0(iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 1, 0), (y, 0, 0.5)\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_0(i) but not the corresponding I-T_0 space.

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.7, 0), (y, 0, 0.7)\}$. For $\alpha = 0.6$, we see that the IFTS (X, t) is α-IF-T_0(ii) but not the corresponding I-T_0 space.

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.5)\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_0(iii) but not the corresponding I-T_0 space.

2.4 Productivity in Intuitionistic Fuzzy T_0-Spaces:

Theorem 2.4.1 Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_0(i) if each IFTS (X_m, t_m) is IF-T_0(i).

Proof: Suppose the IFTS (X_m, t_m) is IF-T_0(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is IF-T_0(i). Choose $x, y \in X$, $x \neq y$. Let $x = \prod x_m$, $y = \prod y_m$. Then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_0(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j}) \in t$ such that $\{\mu_{A_j}(x_j) = 1, \nu_{A_j}(x_j) = 0;$ $\mu_{A_j}(y_j) = 0, \nu_{A_j}(y_j) = 1\}$ or $\{\mu_{A_j}(y_j) = 1, \nu_{A_j}(y_j) = 0;$ $\mu_{A_j}(x_j) = 0,$ $\nu_{A_j}(x_j) = 1\}$. Suppose $\mu_{A_j}(x_j) = 1$, $\nu_{A_j}(x_j) = 0$; $\mu_{A_j}(y_j) = 0$, $\nu_{A_j}(y_j) = 1$. Now consider the basic IFOSs $\prod A_k \in \prod t_k$ where $A_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$ when $k = j$. Then $\prod A_k(x) = (\inf_{k \in J} \mu_{A_k}(x_k), \sup_{k \in J} \nu_{A_k}(x_k)) = (1, 0)$; $\prod A_k(y) = (\inf_{k \in J} \mu_{A_k}(y_k), \sup_{k \in J} \nu_{A_k}(y_k)) = (0, 1)$. Hence (X, t) is IF-T_0(i).

For n = ii, iii, iv, it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_0(n) if each IFTS (X_m, t_m) is IF-T_0(n).

Theorem: 2.4.2 Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-T_0(i) if each IFTS (X_m, t_m) is α-IF-T_0(i).

Proof: Suppose the IFTS (X_m, t_m) is α-IF-T_0(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is α-IF-T_0(i). Let $\alpha \in (0, 1)$. Choose $x, y \in X$, $x \neq y$. Let $x = \prod x_m$, $y = \prod y_m$. Then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is α-IF-T_0(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j}) \in t$ such that $\{ \mu_{A_j}(x_j) = 1,\ \nu_{A_j}(x_j) = 0;\ \mu_{A_j}(y_j) = 0,\ \nu_{A_j}(y_j) \geq \alpha \}$ or $\{ \mu_{A_j}(y_j) = 1,\ \nu_{A_j}(y_j) = 0;\ \mu_{A_j}(x_j) = 0,\ \nu_{A_j}(x_j) \geq \alpha \}$. Suppose $\mu_{A_j}(x_j) = 1,\ \nu_{A_j}(x_j) = 0;\ \mu_{A_j}(y_j) = 0,\ \nu_{A_j}(y_j) \geq \alpha$. Now consider the basic IFOSs $\prod A_k \in \prod t_k$ where $A_k = (1^{\sim}, 0^{\sim})$ for $k \in J$, $k \neq j$ and $A_k = A_j$ when k = j.Then $A(x) = (\mu_A, \nu_A)(x) = \prod A_k(x) = (\inf_{k\in J} \mu_{A_k}(x_k),\ \sup_{k\in J} \nu_{A_k}(x_k))$ and $A(y) = (\mu_A, \nu_A)(y) = \prod A_k(y) = (\inf_{k\in J} \mu_{A_k}(y_k),\ \sup_{k\in J} \nu_{A_k}(y_k))$. Now, $\{ \mu(x) = (\inf_{k\in J} \mu_{A_k}(x_k) = 1,\ \nu(x) = \sup_{k\in J} \nu_{A_k}(x_k) = 0\}$; $\{\mu(y) = \inf_{k\in J} \mu_{A_k}(y_k) = 0;\ \nu(y) = \sup_{k\in J} \nu_{A_k}(y_k) \geq \alpha\}$. Hence (X, t) is α-IF-$T_0$(i).

For n = ii, iii, it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-T_0((n) if each IFTS (X_m, t_m) is α-IF-T_0(n).

2.5 Mappings in Intuitionistic Fuzzy T_0 -spaces:

Theorem 2.5.1 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_0(i) $\Leftrightarrow$ (Y, s) is IF-T_0(i).

(2) (X, t) is IF-T_0(ii) $\Leftrightarrow$ (Y, s) is IF-T_0(ii).

(3) (X, t) is IF-T_0(iii) $\Leftrightarrow$ (Y, s) is IF-T_0(iii).

(4) (X, t) is IF-T_0(iv) $\Leftrightarrow$ (Y, s) is IF-T_0(iv).

Proof(1): Suppose the IFTS (X, t) is IF-T_0(i). We shall prove that the IFTS (Y, s) is IF-T_0(i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since (X, t) is IF-T_0(i), then there exists $A = (\mu_A, \nu_A) \in t$ such that $\{\mu_A(x_1) = 1, \nu_A(x_1) = 0; \mu_A(x_2) = 0, \nu_A(x_2) = 1\}$ or $\{\mu_A(x_2) = 1, \nu_A(x_2) = 0; \mu_A(x_1) = 0, \nu_A(x_1) = 1\}$. Suppose $\mu_A(x_1) = 1, \nu_A(x_1) = 0; \mu_A(x_2) = 0, \nu_A(x_2) = 1$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1, (f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0\}$; $\{(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2) = 0, (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2) = 1\}$. Since f is IF-continuous, then $(f(\mu_A), f(\nu_A)) \in s$ with $(f(\mu_A))(y_1) = 1, (f(\nu_A))(y_1) = 0; (f(\mu_A))(y_2) = 0, (f(\nu_A))(y_2) = 1$. Therefore, the IFTS (Y, s) is IF-T_0(i).

Conversely, suppose the IFTS (Y, s) is IF-T_0(i). We shall prove that the IFTS (X, t) is IF-T_0(i). Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is one-one, then there exists $y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. Again, since (Y, s) is IF-T_0(i), then there exists $B = (\mu_B, \nu_B) \in s$ such that $\{\mu_B(y_1) = 1, \nu_B(y_1) = 0; \mu_B(y_2) = 0, \nu_B(y_2) = 1\}$ or $\{\mu_B(y_2) = 1, \nu_B(y_2) = 0; \mu_B(y_1) = 0, \nu_B(y_1) = 1\}$. Suppose $\mu_B(y_1) = 1$, $\nu_B(y_1) = 0; \mu_B(y_2) = 0, \nu_B(y_2) = 1$. Now, $\{(f^{-1}(\mu_B))(x_1) = \mu_B(f(x_1)) = \mu_B(y_1) = 1, (f^{-1}(\nu_B))(x_1) = \nu_B(f(x_1)) = \nu_B(y_1) = 0\}; \{(f^{-1}(\mu_B))(x_2) = \mu_B(f(x_2)) = \mu_B(y_2) = 0, (f^{-1}(\nu_B))(x_2) = \nu_B(f(x_2)) = \nu_B(y_2) = 1\}$. Since f is IF-continuous, then $(f^{-1}(\mu_B), f^{-1}(\nu_B)) \in t$ with $f^{-1}(\mu_B))(x_1) = 1$, $\left(f^{-1}(\nu_B)\right)(x_1) = 0; (f^{-1}(\mu_B))(x_2) = 0, (f^{-1}(\nu_B))(x_2) = 1$. Therefore, the IFTS (X, t) is IF-T_0(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 2.5.2 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(a) (X, t) is α-IF-T_0(i) $\Leftrightarrow$ (Y, s) is α-IF-T_0(i).

(b) (X, t) is α-IF-T_0(ii) $\Leftrightarrow$ (Y, s) is α-IF-T_0(ii).

(c) (X, t) is α-IF-T_0(iii) $\Leftrightarrow$ (Y, s) is α-IF-T_0(iii).

Proof (a): Suppose the IFTS (X, t) is α-IF-T_0(i). We shall prove that the IFTS (Y, s) is α-IF-T_0(i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since (X, t) is α-IF-T_0(i), then there exists $A = (\mu_A, \nu_A) \in t$ such that $\{\mu_A(x_1) = 1, \nu_A(x_1) = 0; \mu_A(x_2) = 0, \nu_A(x_2) \geq \alpha\}$ or $\{\mu_A(x_2) = 1,$

$\nu_A(x_2) = 0;\ \ \mu_A(x_1) = 0,\ \ \nu_A(x_1) \geq \alpha\}$. Suppose $\mu_A(x_1) = 1,\ \ \nu_A(x_1) = 0;$ $\mu_A(x_2) = 0,\ \ \nu_A(x_2) \geq \alpha$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1,$ $(f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0\}$; $\{(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) =$ $\mu_A(x_2) = 0,\ \ (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2) \geq \alpha\}$. Since f is IF-continuous, then $(f(\mu_A), f(\nu_A)) \in s$ with $(f(\mu_A))(y_1) = 1,\ (f(\nu_A))(y_1) = 0;$ $(f(\mu_A))(y_2) = 0,\ (f(\nu_A))(y_2) \geq \alpha$. Therefore, the IFTS $(Y,\ s)$ is α-IF-T_0(i).

Conversely, suppose the IFTS $(Y,\ s)$ is α-IF-T_0(i). We shall prove that the IFTS $(X,\ t)$ is α-IF-T_0(i). Let $x_1,\ x_2 \in X$ with $x_1 \neq x_2$.Since f is one-one, then there exists $y_1,\ y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. Again, since $(Y,\ s)$ is α-IF-T_0(i), then there exists $B = (\mu_B, \nu_B) \in s$ such that $\{\mu_B(y_1) = 1,\ \ \nu_B(y_1) = 0;\ \ \mu_B(y_2) = 0,$ $\nu_B(y_2) \geq \alpha\}$ or $\{\mu_B(y_2) = 1,\ \ \nu_B(y_2) = 0;\ \ \mu_B(y_1) = 0,\ \ \nu_B(y_1) \geq \alpha\}$. Suppose $\mu_B(y_1) = 1$, $\nu_B(y_1) = 0$; $\mu_B(y_2) = 0$, $\nu_B(y_2) \geq \alpha$. Now, $\{(f^{-1}(\mu_B))(x_1) = \mu_B(f(x_1)) = \mu_B(y_1) = 1,\ \ (f^{-1}(\nu_B))(x_1) = \nu_B(f(x_1)) =$ $\nu_B(y_1) = 0\}$; $\{(f^{-1}(\mu_B))(x_2) = \mu_B(f(x_2)) = \mu_B(y_2) = 0,\ \ (f^{-1}(\nu_B))(x_2) =$ $\nu_B(f(x_2)) = \nu_B(y_2) \geq \alpha\}$. Since f is IF-continuous, then $(f^{-1}(\mu_B), f^{-1}(\nu_B)) \in t$ with $(f^{-1}(\mu_B))(x_1) = 1,$ $\left(f^{-1}(\nu_B)\right)(x_1) = 0;$ $(f^{-1}(\mu_B))(x_2) = 0,$ $(f^{-1}(\nu_B))(x_2) \geq \alpha$. Therefore, the IFTS $(X,\ t)$ is α-IF-T_0(i).

(b) and (c) can be proved in the similar way.

CHAPTER 3

On Intuitionistic Fuzzy T_1-Spaces

Lal and Srivastava[127] established the concepts of T_1-space in fuzzy topological spaces. Hossain and Ali[57] studied T_1-space in fuzzy topological spaces. Shen[118] introduced T_1-space in fuzzifying topology. Li[72] discussed T_1-space in L-fuzzy topological spaces. Yue and Fang[149] considered the separation axioms T_1-space in an intuitionistic fuzzy (I-fuzzy) topological spaces.

In this chapter, we introduce seven possible notions of intuitionistic fuzzy T_1 (in short, IF-T_1) space. We give several characterizations of these notions and discuss certain relationship among them. We study that all these notions satisfy "good extension" property. It is shown that these notions are hereditary and productive. We observe that all concepts are preserved under one-one, onto and continuous mappings.

3.1 Definition and Properties:

Definition 3.1.1 An intuitionistic fuzzy topological space (X, t) is called

(1) IF-T_1(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) = 1$.

(2) IF-T_1(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) > 0$.

(3) IF-T_1(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) > 0$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) = 1$.

(4) IF-T_1(iv) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) > 0$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) > 0$.

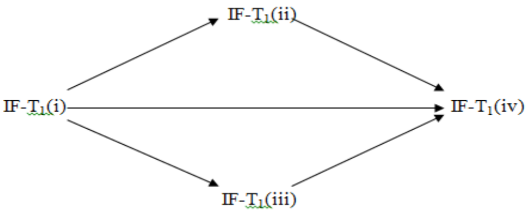
Definition 3.1.2 Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α -IF-T_1(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

(b) α -IF-T_1(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

(c) α -IF-T_1(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) > 0$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

Theorem 3.1.3 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is IF-T_1(i). We shall prove that (X, t) is IF-T_1(ii). Since (X, t) is IF-T_1(i), then for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) = 1, \nu_B(y) = 0;\ \mu_B(x) = 0,\ \nu_B(x) = 1 \implies \mu_A(x) = 1,\ \nu_A(x) = 0;$ $\mu_A(y) = 0,\ \nu_A(y) > 0$ and $\mu_B(y) = 1,\ \nu_B(y) = 0;\ \mu_B(x) = 0,\ \nu_B(x) > 0.$ Which is IF-T_1(ii). Hence IF-T_1(i) $\implies$ IF-T_1(ii).

Again, suppose (X, t) is IF-T_1(i). We shall prove that (X, t) is IF-T_1(iii). Since (X, t) is IF-T_1(i), then for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) = 1$ and $\mu_B(y) = 1,\ \nu_B(y) = 0;\ \mu_B(x) = 0, \nu_B(x) = 1 \implies \mu_A(x) > 0,\ \nu_A(x) = 0;$ $\mu_A(y) = 0,\ \nu_A(y) = 1$ and $\mu_B(y) > 0,\ \nu_B(y) = 0;\ \mu_B(x) = 0,\ \nu_B(x) = 1.$ Which is IF-T_1(iii). Hence IF-T_1(i) $\implies$ IF-T_1(iii).

Furthermore, it can prove that IF-T_1(i) $\implies$ IF-T_1(iv), IF-T_1(ii) $\implies$ IF-T_1(iv) and IF-T_1(iii) $\implies$ IF-T_1(iv).

None of the reverse implications is true in general which can be seen from the following examples.

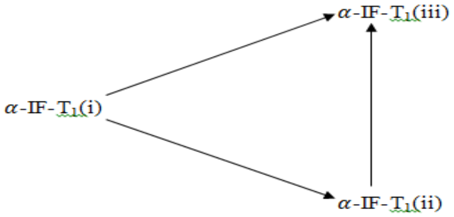
Example (a) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 1, 0), (y, 0, 0.5)} and B = {(x, 0, 0.7), (y, 1, 0)}. We see that the IFTS (X, t) is IF-T_1(ii) but not IF-T_1(i).

Example (b) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.2, 0), (y, 0, 1)} and B = {(x, 0, 1), (y, 0.6, 0)}. We see that the IFTS (X, t) is IF-T_1(iii) but not IF-T_1(i).

Example (c) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 1, 0), (y, 0, 0.4)} and B = {(x, 0, 0.6), (y, 1, 0)}. We see that the IFTS (X, t) is IF-T_1(ii) but not IF-T_1(iii).

Example (d) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.3, 0), (y, 0, 1)} and B = {(x, 0, 1), (y, 0.5, 0)}. We see that the IFTS (X, t) is IF-T_1(iii) but not IF-T_1(ii).

Theorem 3.1.4 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is α-IF-T_1(i). We shall prove that (X, t) is α-IF-T_1(i). Let $\alpha \in (0, 1)$. Since (X, t) is α-IF-T_1(i), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ $\Rightarrow$ $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ for any $\alpha \in (0, 1)$. Which is α-IF-T_1(ii). Hence α-IF-T_1(i) $\Rightarrow$ α-IF-T_1(ii).

Again, suppose (X, t) is α-IF-T_1(ii) space. We shall prove that (X, t) is α-IF-T_1(iii). Let $\alpha \in (0, 1)$. Since (X, t) is α-IF-T_1(ii), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ $\Rightarrow$ $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) > 0$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ for any $\alpha \in (0, 1)$. Which is α-IF-T_1(iii). Hence α-IF-T_1(ii) $\Rightarrow$ α-IF-T_1(iii).

Furthermore, it can prove that α-IF-T_1(i) $\Rightarrow$ α-IF-T_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.5, 0), (y, 0, 0.5)} and B = {(x, 0, 0.4), (y, 0.4, 0)}. For $\alpha = 0.3$, we see that the IFTS (X, t) is α-IF-T_1(ii) but not α-IF-T_1(i).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.4)\}$ and $B = \{(x, 0, 0.5), (y, 0.3, 0)\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_1(iii) but neither α-IF-T_1(ii) nor α-IF-T_1(i).

Theorem 3.1.5 Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then

(1) β-IF-T_1(i) $\Longrightarrow$ α-IF-T_1(i).

(2) β-IF-T_1(ii) $\Longrightarrow$ α-IF-T_1(ii).

(3) β-IF-T_1(iii) $\Longrightarrow$ α-IF-T_1(iii).

Proof (1): Suppose the intuitionistic fuzzy topological space (X, t) is β-IF-T_1(i). We shall prove that (X, t) is α-IF-T_1(i). Since (X, t) is β-IF-T_1(i), then for all $x, y \in X$, $x \neq y$ with $\beta \in (0, 1)$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \beta$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \beta \Longrightarrow \mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ as $0 < \alpha \leq \beta < 1$, which is α-IF-T_1(i). Hence β-IF-T_1(i) $\Longrightarrow$ α-IF-T_1(i).

Furthermore, it can prove that β-IF-T_1(ii) $\Longrightarrow$ α-IF-T_1(ii) and β-IF-T_1(iii) $\Longrightarrow$ α-IF-T_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 0.6)\}$ and $B = \{(x, 0, 0.5),$

(y, 1, 0)}. For $\alpha = 0.5$ and $\beta = 0.7$, we see that the IFTS (X, t) is α-IF-T_1(i) but not β-IF-T_1(i).

Example (b) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.5, 0), (y, 0, 0.5)} and B = {(x, 0, 0.6), (y, 0.7, 0)}. For $\alpha = 0.5$ and $\beta = 0.8$, we see that the IFTS (X, t) is α-IF-T_1(ii) but not β-IF-T_1(ii).

Example (c) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.2, 0), (y, 0, 0.4)} and B= {(x, 0, 0.5), (y, 0.3, 0)}. For $\alpha = 0.4$ and $\beta = 0.6$, we see that the IFTS (X, t) is α-IF-T_1(iii) but not β-IF-T_1(iii).

3.2 Subspaces:

Theorem 3.2.1 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$, then

(1) (X, t) is IF-T_1(i) $\Rightarrow$ (U, t_U) is IF-T_1(i).

(2) (X, t) is IF-T_1(ii) $\Rightarrow$ (U, t_U) is IF-T_1(ii).

(3) (X, t) is IF-T_1(iii) $\Rightarrow$ (U, t_U) is IF-T_1(iii).

(4) (X, t) is IF-T_1(iv) $\Rightarrow$ (U, t_U) is IF-T_1(iv).

Proof (1): Suppose (X, t) is IF-T_1(i). We shall prove that (U, t_U) is IF-T_1(i). Let $x, y \in U$ with $x \neq y$ then $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Since (X, t) is IF-T_1(i) then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that

$\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) = 1 \implies (\mu_A|U)(x) = 1$, $(\nu_A|U)(x) = 0$; $(\mu_A|U)(y) = 0$; $(\nu_A|U)(y) = 1$ and $(\mu_B|U)(y) = 1$, $(\nu_B|U)(y) = 0$; $(\mu_B|U)(x) = 0$, $(\nu_B|U)(x) = 1$. Hence $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U \implies (B|U, C|U) \in t_U$. Therefore, the intuitionistic fuzzy topological space (U, t_U) is IF-T_1(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 3.2.2 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ and let $\alpha \in (0, 1)$, then

(a) (X, t) is α-IF-T_1(i) $\implies$ (U, t_U) is α-IF-T_1(i).

(b) (X, t) is α-IF-T_1(ii) $\implies$ (U, t_U) is α-IF-T_1(ii).

(c) (X, t) is α-IF-T_1(iii) $\implies$ (U, t_U) is α-IF-T_1(iii).

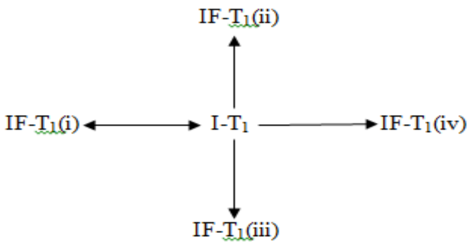
Proof (a): Suppose (X, t) is α-IF-T_1(i). We shall prove that (U, t_U) is α-IF-T_1(i). Let $x, y \in U$, $x \neq y$ then $x, y \in X$, $x \neq y$ as $U \subseteq X$. Since (X, t) is α-IF-T_1(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha \implies (\mu_A|U)(x) = 1$, $(\nu_A|U)(x) = 0$; $(\mu_A|U)(y) = 0$, $(\nu_A|U)(y) \geq \alpha$ and $(\mu_B|U)(y) = 1$, $(\nu_B|U)(y) = 0$; $(\mu_B|U)(x) = 0$, $(\nu_B|U)(x) \geq \alpha$. Hence $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U \implies (B|U, C|U) \in t_U$. Therefore, the intuitionistic fuzzy topological space (U, t_U) is α-IF-T_1(i).

(b) and (c) can be proved in the similar way.

3.3 Good extension:

Definition 3.3.1 An intuitionistic topological space (X, τ) is called intuitionistic T_1-space (I-T_1 space) if for all $x,\ y \in X,\ \ x \neq y$ there exists $C = (C_1,\ C_2)$, $D = (D_1,\ D_2) \in \tau$ such that $(x \in C_1,\ x \notin C_2;\ y \notin C_1,\ y \in C_2)$ and $(y \in D_1,\ y \notin D_2;\ x \notin D_1,\ x \in D_2)$.

Theorem 3.3.2 Let $(X,\ \tau)$ be an intuitionistic topological space and $(X,\ t)$ be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose $(X,\ \tau)$ is I-T_1. We shall prove that $(X,\ t)$ is IF-T_1(i). Since $(X,\ \tau)$ is I-T_1, then for all $x,\ y \in X,\ \ x \neq y$ there exists $C = (C_1,\ C_2)$, $D = (D_1,\ D_2) \in \tau$ such that $x \in C_1, x \notin C_2;\ y \notin C_1,\ y \in C_2$ and $y \in D_1$, $y \notin D_2;\ x \notin D_1,\ x \in D_2 \Longrightarrow 1_{C_1}(x) = 1, 1_{C_2}(x) = 0; 1_{C_1}(y) = 0,\ 1_{C_2}(y) = 1$ and $1_{D_1}(y) = 1,\ \ 1_{D_2}(y) = 0;\ \ 1_{D_1}(x) = 0,\ \ 1_{D_2}(x) = 1$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A,\ \ 1_{D_1} = \mu_B,\ \ 1_{D_2} = \upsilon_B$ then $\mu_A(x) = 1,\ \ \upsilon_A(x) = 0;\ \ \mu_A(y) = 0$, $\upsilon_A(y) = 1$ and $\mu_B(y) = 1,\ \ \upsilon_B(y) = 0;\ \ \mu_B(x) = 0,\ \ \upsilon_B(x) = 1$. Hence $\{(\mu_A, \upsilon_A),\ (\mu_B, \upsilon_B)\} \in t \Longrightarrow (X,\ t)$ is IF-T_1. Therefore I-$T_1 \Longrightarrow$ IF-T_1(i).

Conversely, suppose (X, t) is IF-T_1(i). We shall prove that (X, τ) is I-T_1. Since (X, t) is IF-T_1(i), then for all x, y $\in$ X, $x \neq y$ there exists $(1_{C_1}, 1_{C_2})$, $(1_{D_1}, 1_{D_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1$ and $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$; $1_{D_1}(x) = 0$, $1_{D_2}(x) = 1 \Longrightarrow x \in C_1$, $x \notin C_2$; $y \notin C_1$, $y \in C_2$ and $y \in D_1$, $y \notin D_2$; $x \notin D_1$, $x \in D_2$. Hence$\{(C_1, C_2), (D_1, D_2)\} \in \tau$ $\Longrightarrow$ (X, τ) is I-T_1. Hence IF-T_1(i) $\Longrightarrow$ I-T_1. Therefore I-T_1 $\Leftrightarrow$ IF-T_1(i).

Furthermore, it can prove that I-T_1 $\Longrightarrow$ IF-T_1(ii), I-T_1 $\Longrightarrow$ IF-T_1(iii) and I-T_1 $\Longrightarrow$ IF-T_1(iv).

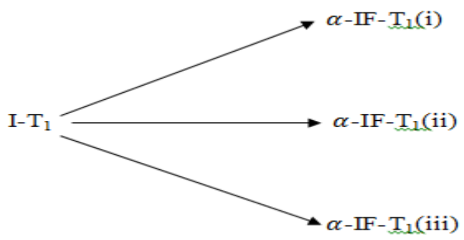
None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 1, 0), (y, 0, 0.4)} and B = {(x, 0, 0.5), (y, 1, 0)}, we see that the IFTS (X, t) is IF-T_1(ii) but not corresponding I-T_1.

Example (b) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.5, 0), (y, 0, 1)} and B = {(x, 0, 1), (y, 0.6, 0)}, we see that the IFTS (X, t) is IF-T_1(iii) but not corresponding I-T_1.

Example (c) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.2, 0), (y, 0, 0.3)} and B = {(x, 0, 0.4), (y, 0.6, 0)}, we see that the IFTS (X, t) is IF-T_1(iv) but not corresponding I-T_1.

Theorem 3.3.3 Let (X, τ) be an intuitionistic topological space and (X, t) be the intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is I-T_1. We shall prove that (X, t) is IF-T_1(i). Let $\alpha \in (0,\ 1)$. Since (X, τ) is I-T_1, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $x \notin C_2$; $y \notin C_1$, $y \in C_2$ and $y \in D_1, y \notin D_2$; $x \notin D_1, x \in D_2 \Longrightarrow 1_{C_1}(x) = 1, 1_{C_2}(x) = 0; 1_{C_1}(y) = 0,$ $1_{C_2}(y) = 1$ and $1_{D_1}(y) = 1,\ 1_{D_2}(y) = 0;\ 1_{D_1}(x) = 0,\ 1_{D_2}(x) = 1$. Let $1_{C_1} = \mu_A,\ 1_{C_2} = \upsilon_A,\ 1_{D_1} = \mu_B,\ 1_{D_2} = \upsilon_B$ then $\mu_A(x) = 1,\ \upsilon_A(x) = 0;$ $\mu_A(y) = 0,\ \upsilon_A(y) = 1$ and $\mu_B(y) = 1,\ \upsilon_B(y) = 0;\ \mu_B(x) = 0,\ \upsilon_B(x) = 1 \Longrightarrow$ $\mu_A(x) = 1,\ \upsilon_A(x) = 0;\ \mu_A(y) = 0,\ \upsilon_A(y) \geq \alpha$ and $\mu_B(y) = 1,\ \upsilon_B(y) = 0;$ $\mu_B(x) = 0,\ \upsilon_B(x) \geq \alpha$ for any $\alpha \in (0,\ 1)$. Hence $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t$ $\Longrightarrow$ (X, t) is α-IF-T_1. Therefore, I-T_1 $\Longrightarrow$ α-IF-T_1(i).

Furthermore, it can prove that I-T_1 $\Longrightarrow$ α-IF-T_1(ii) and I-T_1 $\Longrightarrow$ α-IF-T_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 1, 0), (y, 0, 0.7)} and B = {(x, 0, 0.8), (y, 1, 0)}. For $\alpha = 0.7$, we see that the IFTS (X, t) is α-IF-T_1(i) but not corresponding I-T_1.

Example (b) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.5, 0), (y, 0, 0.5)} and B = {(x, 0, 0.6), (y, 0.6, 0)}. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_1(ii) but not corresponding I-T_1.

Example (c) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 0.3, 0), (y, 0, 0.5)} and B = {(x, 0, 0.5), (y, 0.4, 0)}. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_1(iii) but not corresponding I-T_1.

3.4 Productivity in Intuitionistic Fuzzy T_1-Spaces:

Theorem 3.4.1 Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological space and let (X, t) be their product of IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_1(i) if each IFTS (X_m, t_m) is IF-T_1(i).

Proof: Suppose the IFTS (X_m, t_m) is IF-T_1(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is IF-T_1(i). Choose $x, y \in X, x \neq y$. Let $x = \prod x_m$, $y = \prod y_m$, then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_1(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$, $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $\{ \mu_{A_j}(x_j) = 1, \ \nu_{A_j}(x_j) = 0 ; \ \mu_{A_j}(y_j) = 0 , \ \nu_{A_j}(y_j) = 1 \}$ and $\{ \mu_{B_j}(y_j) = 1,$

$\nu_{B_j}(y_j) = 0;\ \mu_{B_j}(x_j) = 0, \nu_{B_j}(x_j) = 1\}$. Now consider the basic IFOSs $\prod A_k$ and $\prod B_k$ where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then $\{\prod A_k(x) = (\inf_{k\in J} \mu_{A_k}(x_k), \sup_{k\in J} \nu_{A_k}(x_k)) = (1, 0)$; $\prod A_k(y) = (\inf_{k\in J} \mu_{A_k}(y_k), \sup_{k\in J} \nu_{A_k}(y_k)) = (0, 1)\}$ and $\{\prod B_k(y) = (\inf_{k\in J} \mu_{B_k}(y_k),$ $\sup_{k\in J} \nu_{B_k}(y_k)) = (1, 0)$; $\prod B_k(x) = (\inf_{k\in J} \mu_{B_k}(x_k), \sup_{k\in J} \nu_{B_k}(x_k)) = (0, 1)\}$. Hence (X, t) is IF-T_1(i).

For n = ii, iii, iv, it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_1(n) if each IFTS (X_m, t_m) is IF-T_1(n).

Theorem 3.4.2 Let $\{(X_m, t_m) : m \in J\}$be a family of IFTS and (X, t) be their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-T_1(i) if each IFTS (X_m, t_m) is α-IF-T_1(i).

Proof: Suppose the IFTS (X_m, t_m) is α-IF-T_1(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is α-IF-T_1(i). Choose $x, y \in X$, $x \neq y$. Let $x = \prod x_m$, $y = \prod y_m$, then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is α-IF-T_1(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$, $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $\{\mu_{A_j}(x_j) = 1,\ \nu_{A_j}(x_j) = 0;\ \mu_{A_j}(y_j) = 0,\ \nu_{A_j}(y_j) \geq \alpha\}$ and $\{\mu_{B_j}(y_j) = 1,\ \nu_{B_j}(y_j) = 0;\ \mu_{B_j}(x_j) = 0, \nu_{B_j}(x_j) \geq \alpha\}$. Now consider the basic IFOSs $\prod A_k$ and $\prod B_k$ where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then $\{A(x) = (\mu_A, \nu_A)(x) = \prod A_k(x) =$ $(\inf_{k\in J} \mu_{A_k}(x_k), \sup_{k\in J} \nu_{A_k}(x_k))$; $A(y) = (\mu_A, \nu_A)(y) = \prod A_k(y) = (\inf_{k\in J} \mu_{A_k}(y_k),$

$\sup_{k\in J} \nu_{A_k}(y_k))\}$ and $\{B(x) = (\mu_B, \nu_B)(x) = \prod B_k(x) = (\inf_{k\in J} \mu_{B_k}(x_k), \sup_{k\in J} \nu_{B_k}(x_k))$; $B(y) = (\mu_B, \nu_B)(y) = \prod B_k(y) = (\inf_{k\in J} \mu_{B_k}(y_k), \sup_{k\in J} \nu_{B_k}(y_k))\}$. Now $\{ \mu_A(x) = \inf_{k\in J} \mu_{A_k}(x_k) = 1, \nu_A(x) = \sup_{k\in J} \nu_{A_k}(x_k) = 0; \mu_A(y) = \inf_{k\in J} \mu_{A_k}(y_k) = 0, \nu_A(y) = \sup_{k\in J} \nu_{A_k}(y_k) \geq \alpha \}$ and $\{ \mu_B(y) = \inf_{k\in J} \mu_{B_k}(y_k) = 1, \nu_B(y) = \sup_{k\in J} \nu_{B_k}(y_k) = 0; \mu_B(x) = \inf_{k\in J} \mu_{B_k}(x_k) = 0, \nu_B(x) = \sup_{k\in J} \nu_{B_k}(x_k) \geq \alpha\}$. Hence (X, t) is α-IF-$T_1$(i).

For n = ii, iii it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_1(n) if each IFTS (X_m, t_m) is IF-T_1(n).

3.5 Mappings in Intuitionistic Fuzzy T_1-spaces:

Theorem 3.5.1 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and f : X → Y be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_1(i) ⇔ (Y, s) is IF-T_1(i).

(2) (X, t) is IF-T_1(ii) ⇔ (Y, s) is IF-T_1(ii).

(3) (X, t) is IF-T_1(iii) ⇔ (Y, s) is IF-T_1(iii).

(4) (X, t) is IF-T_1(iv) ⇔ (Y, s) is IF-T_1(iv).

Proof: Suppose the IFTS (X, t) is IF-T_1(i). We shall prove that the IFTS (Y, s) is IF-T_1(i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then ∃ $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since (X, t) is IF-T_1(i), then there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that

$(\mu_A(x_1) = 1, \ \nu_A(x_1) = 0; \ \mu_A(x_2) = 0, \nu_A(x_2) = 1)$ and $(\mu_B(x_2) = 1, \nu_B(x_2) = 0; \ \mu_B(x_1) = 0, \nu_B(x_1) = 1)$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1, \ (f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0; (f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2) = 0, \ (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2) = 1\}$ and $\{(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1, \ (f(\nu_B))(y_2) = \nu_B(f^{-1}(y_2)) = \nu_B(x_2) = 0; (f(\mu_B))(y_1) = \mu_B(f^{-1}(y_1)) = \mu_B(x_1) = 0, \ (f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) = 1\}$. Since f is IF-continuous, then $\{(f(\mu_A), f(\nu_A)), (f(\mu_B), f(\nu_B))\} \in s$ with $\{\big(f(\mu_A)\big)(y_1) = 1, \big(f(\nu_A)\big)(y_1) = 0, \big(f(\mu_A)\big)(y_2) = 0, \big(f(\nu_A)\big)(y_2) = 1\}$ and $\{\big(f(\mu_B)\big)(y_2) = 1, \ \big(f(\nu_B)\big)(y_2) = 0; \ \big(f(\mu_B)\big)(y_1) = 0, \big(f(\nu_B)\big)(y_1) = 1$. Therefore, the IFTS (Y, s) is IF-T_1(i).

Conversely, suppose the IFTS (Y, s) is IF-T_1(i). We shall prove that the IFTS (X, t) is IF-T_1(i). Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is one-one, then $\exists \ y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. Since (Y, s) is IF-T_1(i), then there exists $C = (\mu_C, \nu_C)$, $D = (\mu_D, \nu_D) \in s$ such that $(\mu_C(y_1) = 1, \ \nu_C(y_1) = 0; \ \mu_C(y_2) = 0, \nu_C(y_2) = 1)$ and $(\mu_D(y_2) = 1, \ \nu_D(y_2) = 0; \ \mu_D(y_1) = 0, \ \nu_D(y_1) = 1)$. Now, $\{(f^{-1}(\mu_C))(x_1) = \mu_C(f(x_1)) = \mu_C(y_1) = 1, \ (f^{-1}(\nu_C))(x_1) = \nu_C(f(x_1)) = \nu_C(y_1) = 0; (f^{-1}(\mu_C))(x_2) = \mu_C(f(x_2)) = \mu_C(y_2) = 0, \ (f^{-1}(\nu_C))(x_2) = \nu_C(f(x_2)) = \nu_C(y_2) = 1\}$ and $\{(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1, (f^{-1}(\nu_D))(x_2) = \nu_D(f(x_2)) = \nu_D(y_2) = 0; (f^{-1}(\mu_D))(x_1) = \mu_D(f(x_1)) = \mu_D(y_1) = 0, (f^{-1}(\nu_D))(x_1) = \nu_D(f(x_1)) = \nu_D(y_1) = 1\}$. Since f is IF-continuous, then $\{(f^{-1}(\mu_C), f^{-1}(\nu_C)), \ (f^{-1}(\mu_D), f^{-1}(\nu_D))\} \in t$ with $\{(f^{-1}(\mu_C))(x_1) = 1,$

$(f^{-1}(\nu_C))(x_1) = 0$; $(f^{-1}(\mu_C))(x_2) = 0$, $(f^{-1}(\nu_C))(x_2) = 1$ } and { $(f^{-1}(\mu_D))(x_2) = 1,$ $(f^{-1}(\nu_D))(x_2) = 0;$ $(f^{-1}(\mu_D))(x_1) = 0$, $(f^{-1}(\nu_D))(x_1) = 1\}$. Therefore, the IFTS $(X,\ t)$ is IF-T_1(i).

(2), (3), (4) can be proved in the similar way.

Theorem 3.5.2 Let $(X,\ t)$ and $(Y,\ s)$ be two intuitionistic fuzzy topological spaces and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(a) $(X,\ t)$ is α-IF-T_1(i) ⇔ $(Y,\ s)$ is α-IF-T_1(i).

(b) $(X,\ t)$ is α-IF-T_1(ii) ⇔ $(Y,\ s)$ is α-IF-T_1(ii).

(c) $(X,\ t)$ is α-IF-T_1(iii) ⇔ $(Y,\ s)$ is α-IF-T_1(iii).

Proof (a): Suppose the IFTS $(X,\ t)$ is α-IF-T_1(i). We shall prove that the IFTS $(Y,\ s)$ is α-IF-T_1(i). Let $y_1,\ y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then ∃ $x_1,\ x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since $(X,\ t)$ is α-IF-T_1(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x_1) = 1,\ \nu_A(x_1) = 0;\ \mu_A(x_2) = 0, \nu_A(x_2) \geq \alpha)$ and $(\mu_B(x_2) = 1,$ $\nu_B(x_2) = 0;\ \mu_B(x_1) = 0, \nu_B(x_1) \geq \alpha)$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1,$ $(f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0;$ $(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2) = 0,$ $(f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2) \geq \alpha\}$ and $\{(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1,$ $(f(\nu_B))(y_2) = \nu_B(f^{-1}(y_2)) = \nu_B(x_2) = 0;$ $(f(\mu_B))(y_1) = \mu_B(f^{-1}(y_1)) = \mu_B(x_1) = 0,$ $(f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) \geq \alpha\}$. Since f is IF-continuous, then $\{(f(\mu_A), f(\nu_A)),$

$(f(\mu_B), f(\nu_B))\} \in s$ with $\{(f(\mu_A))(y_1) = 1, (f(\nu_A))(y_1) = 0, (f(\mu_A))(y_2) = 0, (f(\nu_A))(y_2) \geq \alpha\}$ and $\{(f(\mu_B))(y_2) = 1, (f(\nu_B))(y_2) = 0; (f(\mu_B))(y_1) = 0, (f(\nu_B))(y_1) \geq \alpha$. Therefore, the IFTS (Y, s) is α-IF-T_1(i).

Conversely, suppose the IFTS (Y, s) is α-IF-T_1(i). We shall prove that the IFTS (X, t) is α-IF-T_1(i). Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is one-one, then $\exists\ y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. Therefore since (Y, s) is α-IF-T_1(i), then there exists $C = (\mu_C, \nu_C)$, $D = (\mu_D, \nu_D) \in s$ such that $\mu_C(y_1) = 1, \nu_C(y_1) = 0; \mu_C(y_2) = 0, \nu_C(y_2) \geq \alpha$ and $\mu_D(y_2) = 1, \nu_D(y_2) = 0; \mu_D(y_1) = 0, \nu_D(y_1) \geq \alpha$. Now, $\{(f^{-1}(\mu_C))(x_1) = \mu_C(f(x_1)) = \mu_C(y_1) = 1, (f^{-1}(\nu_C))(x_1) = \nu_C(f(x_1)) = \nu_C(y_1) = 0; (f^{-1}(\mu_C))(x_2) = \mu_C(f(x_2)) = \mu_C(y_2) = 0, (f^{-1}(\nu_C))(x_2) = \nu_C(f(x_2)) = \nu_C(y_2) \geq \alpha\}$ and $\{(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1, (f^{-1}(\nu_D))(x_2) = \nu_D(f(x_2)) = \nu_D(y_2) = 0; (f^{-1}(\mu_D))(x_1) = \mu_D(f(x_1)) = \mu_D(y_1) = 0, (f^{-1}(\nu_D))(x_1) = \nu_D(f(x_1)) = \nu_D(y_1) \geq \alpha\}$. Since f is IF-continuous, then $\{(f^{-1}(\mu_C), f^{-1}(\nu_C)), (f^{-1}(\mu_D), f^{-1}(\nu_D))\} \in t$ with $\{(f^{-1}(\mu_C))(x_1) = 1, (f^{-1}(\nu_C))(x_1) = 0; (f^{-1}(\mu_C))(x_2) = 0, (f^{-1}(\nu_C))(x_2) \geq \alpha\}$ and $\{(f^{-1}(\mu_D))(x_2) = 1, (f^{-1}(\nu_D))(x_2) = 0; (f^{-1}(\mu_D))(x_1) = 0, (f^{-1}(\nu_D))(x_1) \geq \alpha\}$. Therefore, the IFTS (X, t) is α-IF-T_1(i).

(b) and (c) can be proved in the similar way.

CHAPTER 4

On Intuitionistic Fuzzy T_2-Spaces

The concepts of Hausdorffness in fuzzy topological spaces were studied by Srivastava[126], Lal[128] and Hossain[55]. Lupianez[76] defined new notions of Hausdorffness in the intuitionistic fuzzy sense and obtained some new properties, in particular in convergence. Bayhan and Coker [20] introduced T_2-space separation axioms in intuitionistic fuzzy topological spaces. Yue and Fang[149] considered the separation axioms T_2-space in an intuitionistic fuzzy (I-fuzzy) topological spaces.

In this chapter, a set of new notions of T_2-properties in intuitionistic fuzzy topological spaces are studied. We introduce seven notions of intuitionistic fuzzy T_2-space. We establish the relationship among them. We also see that all these notions satisfy "good extension" property. Furthermore, it is proved that these notions are hereditary and productive. Moreover, we observe that all concepts are preserved under one-one, onto and continuous mappings.

4.1 Definition and Properties:

Definition 4.1.1 An intuitionistic fuzzy topological space $(X,\ t)$ is called

(1) IF-T_2(i) if for all $x,\ y \in X,\ x \neq y$ there exists $A = (\mu_A, \nu_A),\ B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1,\ \nu_A(x) = 0;\ \mu_B(y) = 1,\ \nu_B(y) = 0$ and $A \cap B = 0_\sim$.

(2) IF-T_2(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) > 0$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$.

(3) IF-T_2(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$.

(4) IF-T_2(iv) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) > 0$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$.

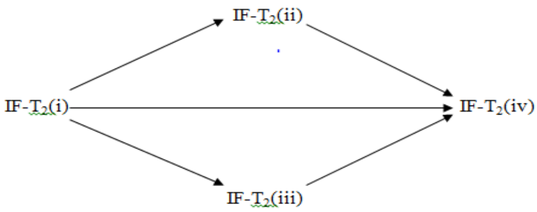
Definition 4.1.2 Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-T_2(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_{\sim}$.

(b) α-IF-T_2(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$.

(c) α-IF-T_2(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$.

Theorem 4.1.3 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is IF-T_2(i). We shall prove that (X, t) is IF-T_2(ii). Since (X, t) is IF-T_2(i), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$, $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ $\Longrightarrow$ $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) > 0$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is IF-T_2(ii) space. Hence IF-T_2(i) $\Longrightarrow$ IF-T_2(ii).

Again, suppose (X, t) is IF-T_2(i). We shall prove that (X, t) is IF-T_2(iii). Since (X, t) is IF-T_2(i), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$, $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ $\Longrightarrow$ $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is IF-T_2(iii). Hence IF-T_2(i) $\Longrightarrow$ IF-T_2(iii).

Furthermore, it can prove that IF-T_2(i) $\Longrightarrow$ IF-T_2(iv), IF-T_2(ii) $\Longrightarrow$ IF-T_2(iv) and IF-T_2(iii) $\Longrightarrow$ IF-T_2(iv).

None of the reverse implications is true in general which can be seen from the following examples.

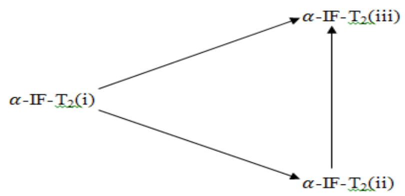
Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 1, 0\}$ and $B = \{y, 0.5, 0\}$. We see that the IFTS (X, t) is IF-T_2(ii) but not IF-T_2(i).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.3, 0\}$ and $B = \{y, 1, 0\}$. We see that the IFTS (X, t) is IF-T_2(iii) but not IF-T_2(i).

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 1, 0\}$ and $B = \{y, 0.7, 0\}$. We see that the IFTS (X, t) is IF-T_2(ii) but not IF-T_2(iii).

Example (d) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.6, 0\}$ and $B = \{y, 1, 0\}$. We see that the IFTS (X, t) is IF-T_2(iii) but not IF-T_2(ii).

Theorem 4.1.4 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is α-IF-T_2(i). We shall prove that (X, t) is α-IF-T_2(ii). Since (X, t) is α-IF-T_2(i), then for all $x, y \in X$, $x \neq y$ there exists

$A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ $\Rightarrow$ $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ for any $\alpha \in (0, 1)$. Which is α-IF-T_2(ii). Hence α-IF-T_2(i) $\Rightarrow$ α-IF-T_2(ii).

Again, suppose (X, t) is α-IF-T_2(ii). We shall prove that (X, t) is α-IF-T_2(iii). Since (X, t) is α-IF-T_2(ii), then for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ $\Rightarrow \mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ for any $\alpha \in (0, 1)$. Which is α-IF-T_2(iii). Hence α-IF-T_2(ii) $\Rightarrow$ α-IF-T_2(iii).

Furthermore, it can prove that α-IF-T_2(i) $\Rightarrow$ α-IF-T_2(iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.3, 0\}$ and $B = \{y, 0.4, 0\}$. For $\alpha = 0.3$, we see that the IFTS (X, t) is α-IF-T_2(ii) but not α-IF-T_2(i)

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.2, 0\}$ and $B = \{y, 0.6, 0\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_2(iii) but neither α-IF-T_2(ii) nor α-IF-T_2(i).

Theorem 4.1.5 Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then

(a) β-IF-T_2(i) $\Longrightarrow$ α-IF-T_2(i).

(b) β-IF-T_2(ii) $\Longrightarrow$ α-IF-T_2(ii).

(c) β-IF-T_2(iii) $\Longrightarrow$ α-IF-T_2(iii).

Proof (a): Suppose the IFTS (X, t) is β-IF-T_2(i). We shall prove that (X, t) is α-IF-T_2(i). Since (X, t) is β-IF-T_2(i), then for all $x, y \in X$, $x \neq y$ with $\beta \in (0, 1)$, there exist $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \beta$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim \Longrightarrow \mu_A(x) = 1, \nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ as $0 < \alpha \leq \beta < 1$. Which is α-IF-$T_2$(i). Hence β-IF-$T_2$(i) $\Longrightarrow$ α-IF-T_2(i).

The proofs β-IF-T_2(ii) $\Longrightarrow$ α-IF-T_2(ii) and β-IF-T_2(iii) $\Longrightarrow$ α-IF-T_2(iii) are similar.

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 1, 0\}$ and $B = \{y, 0.6, 0\}$. For $\alpha = 0.5$ and $\beta = 0.7$, we see that the IFTS (X, t) is α-IF-T_2(i) but not β-IF-T_2(i).

Example (b) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.5, 0\}$ and $B = \{y, 0.4, 0\}$. For $\alpha = 0.4$ and $\beta = 0.8$, we see that the IFTS (X, t) is α-IF-T_2(ii) but not β-IF-T_2(ii).

Example (c) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.4, 0\}$ and $B = \{y, 0.5, 0\}$. For $\alpha = 0.5$ and $\beta = 0.6$, we see that the IFTS (X, t) is α-IF-T_2(iii) but not β-IF-T_2(iii).

4.2 Subspaces:

Theorem 4.2.1 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{A|U : A \in t\}$, then

(1) (X, t) is IF-T_2(i) $\Rightarrow$ (U, t_U) is IF-T_2(i).

(2) (X, t) is IF-T_2(ii) $\Rightarrow$ (U, t_U) is IF-T_2(ii).

(3) (X, t) is IF-T_2(iii) $\Rightarrow$ (U, t_U) is IF-T_2(iii).

(4) (X, t) is IF-T_2(iv) $\Rightarrow$ (U, t_U) is IF-T_2(iv).

Proof (1): Suppose (X, t) is IF-T_2(i). We shall prove that (U, t_U) is IF-T_2(i). Let $x, y \in U$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Since (X, t) is IF-T_2(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ $\Rightarrow$ $(\mu_A|U)(x) = 1$, $(\nu_A|U)(x) = 0$; $(\mu_B|U)(y) = 1$, $(\nu_B|U)(y) = 0$ and $\{(\mu_A|U, \nu_A|U) \cap (\mu_B|U, \nu_B|U)\} = 0_\sim$. Hence $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U \Rightarrow \{A|U, B|U\} \in t_U$. Therefore, the intuitionistic fuzzy topological space (U, t_U) is IF-T_2(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 4.2.2 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ and $\alpha \in (0, 1)$, then

(a) (X, t) is α-IF-T_2(i) $\Longrightarrow$ (U, t_U) is α-IF-T_2(i).

(b) (X, t) is α-IF-T_2(ii) $\Longrightarrow$ (U, t_U) is α-IF-T_2(ii).

(c) (X, t) is α-IF-T_2(iii) $\Longrightarrow$ (U, t_U) is α-IF-T_2(iii).

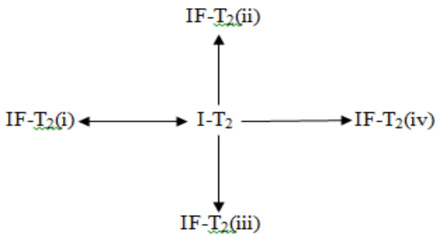
Proof (a): Suppose (X, t) is α-IF-T_2(i). We shall prove that (U, t_U) is α-IF-T_2(i). Let $x, y \in U$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Since (X, t) is α-IF-T_2(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$ $\Longrightarrow$ $(\mu_A|U)(x) = 1$, $(\nu_A|U)(x) = 0$; $(\mu_B|U)(y) \geq \alpha$, $(\nu_B|U)(y) = 0$ and $\{(\mu_A|U, \nu_A|U) \cap (\mu_B|U, \nu_B|U)\} = 0_\sim$. Hence $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U$ $\Longrightarrow \{A|U, B|U\} \in t_U$. Therefore, the intuitionistic fuzzy topological space (U, t_U) is α-IF-T_2(i).

(b) and (c) can be proved the similar way.

4.3 Good Extension:

Definition 4.3.1 An intuitionistic topological space (X, τ) is called intuitionistic T_2-space (I-T_2 space) if for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in D_1$ and $C \cap D = \phi_\sim$.

Theorem 4.3.2 Let (X, τ) be an intuitionistic topological space and (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is I-T_2 space. We shall prove that (X, t) is IF-T_2(i). Since (X, τ) is I-T_2, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in D_1$ and $C \cap D = \phi_\sim \Rightarrow 1_{C_1}(x) = 1$, $1_{D_1}(y) = 1$ and $(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2}) = 0_\sim$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, $1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$, then $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_B(y) = 1$, $\upsilon_B(y) = 0$ and $(\mu_A, \upsilon_A) \cap (\mu_B, \upsilon_B) = 0_\sim$. Hence $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \Rightarrow (X, t)$ is IF-T_2(i). Therefore, I-$T_2 \Rightarrow$ IF-T_2(i).

Conversely, suppose (X, t) is IF-T_2(i). We shall prove that (X, τ) is I-T_2. Since (X, t) is IF-T_2(i), then for all $x, y \in X$, $x \neq y$ there exists $(1_{C_1}, 1_{C_2})$, $(1_{D_1}, 1_{D_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2}) = 0_\sim \Rightarrow x \in C_1$, $x \notin C_2$; $y \in D_1$, $y \notin D_2$ and $C \cap D = \phi_\sim$. Hence $\{(C_1, C_2), (D_1, D_2)\} \in \tau \Rightarrow (X, \tau)$ is I-T_2. Hence IF-T_2(i) $\Rightarrow$ I-T_2. Therefore I-$T_2 \Leftrightarrow$ IF-T_2(i).

Furthermore, it can be shown that I-T_2 $\Rightarrow$ IF-T_2(ii), I-T_2 $\Rightarrow$ IF-T_2(iii) and I-T_2 $\Rightarrow$ IF-T_2(iv).

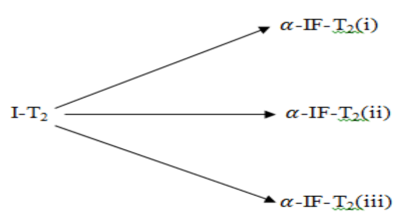
None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 1, 0\}$ and $B = \{y, 0.3, 0\}$, we see that the IFTS (X, t) is IF-T_2(ii) but not corresponding I-T_2 .

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A , B\}$ where $A= \{x, 0.5, 0\}$ and $B= \{y, 1, 0\}$, we see that the IFTS (X, t) is IF-T_2(iii) but not corresponding I-T_2 .

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.2, 0\}$ and $B = \{y, 0.6, 0\}$, we see that the IFTS (X, t) is IF-T_2(iv) but not corresponding I-T_2 .

Theorem 4.3.3 Let (X, τ) be an intuitionistic topological space and (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is I-T_2 space. We shall prove that (X, t) is α-IF-T_2(i). Since (X, τ) is I-T_2, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in D_1$ and $C \cap D = \phi_\sim \implies 1_{C_1}(x) = 1$, $1_{D_1}(y) = 1$ and $(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2}) = 0_\sim \implies 1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2}) = 0_\sim$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, $1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$ then $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_B(y) = 1$, $\upsilon_B(y) = 0$ and $(\mu_A, \upsilon_A) \cap (\mu_B, \upsilon_B) = 0_\sim \implies \mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\upsilon_B(y) = 0$ and $(\mu_A, \upsilon_A) \cap (\mu_B, \upsilon_B) = 0_\sim$ for any $\alpha \in (0, 1)$. Hence $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \implies (X, t)$ is α-IF-T_2(i). Therefore I-$T_2 \implies$ α-IF-T_2(i).

Furthermore, it can be shown that I-$T_2 \implies$ α-IF-T_2(ii) and I-$T_2 \implies$ α-IF-T_2(iii). None of the reverse implications is true in general as can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 1, 0\}$ and $B = \{y, 0.8, 0\}$. For $\alpha = 0.7$, we see that the IFTS (X, t) is α-IF-T_2(i) but not corresponding I-T_2.

Example (b) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.5, 0\}$ and $B = \{y, 0.6, 0\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_2(ii) but not corresponding I-T_2.

Example (c) Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{x, 0.3, 0\}$ and $B = \{y, 0.4, 0\}$. For $\alpha = 0.4$, we see that the IFTS (X, t) is α-IF-T_2(iii) but not corresponding I-T_2.

4.4 Productivity in intuitionistic fuzzy T_2-spaces:

Theorem 4.4.1 Let $\{(X_m, t_m) : m \in J\}$be a family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_2(i) if each IFTS (X_m, t_m) is IF-T_2(i).

Proof: Suppose the IFTS (X_m, t_m) is IF-T_2(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is IF-T_2(i). Choose $x, y \in X, x \neq y$. Let $x = \prod x_m$, $y = \prod y_m$, then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_2(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$ $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $\mu_{A_j}(x_j) = 1$, $\nu_{A_j}(x_j) = 0$; $\mu_{B_j}(y_j) = 1$, $\nu_{B_j}(y_j) = 0$ and $A_j \cap B_j = 0_\sim$. Now consider the basic IFOSs $\prod A_k$ and $\prod B_k$ where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then $\prod A_k(x) = (\inf_{k\in J} \mu_{A_k}(x_k), \sup_{k\in J} \nu_{A_k}(x_k)) = (1, 0)$; $\prod B_k(y) = (\inf_{k\in J} \mu_{B_k}(y_k), \sup_{k\in J} \nu_{B_k}(y_k)) = (1, 0)$ and $\prod A_k \cap \prod B_k = 0_\sim$ because for $j \in J$, $A_j \cap B_j = 0_\sim$. Hence (X, t) is IF-T_2(i).

For n = ii, iii, iv, it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is IF-T_2(n) if each IFTS (X_m, t_m) is IF-T_2(n).

Theorem 4.4.2 Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-T_2(i) if each IFTS (X_m, t_m) is α-IF-T_2(i).

Proof: Suppose the IFTS (X_m, t_m) is α-IF-T_2(i) for all $m \in J$. We shall prove that the product IFTS (X, t) is α-IF-T_2(i). Choose $x, y \in X$, $x \neq y$. Let

$x = \prod x_m$, $y = \prod y_m$, then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is α-IF-T_2(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$, $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $\mu_{A_j}(x_j) = 1$, $\nu_{A_j}(x_j) = 0$; $\mu_{B_j}(y_j) \geq \alpha$, $\nu_{B_j}(y_j) = 0$ and $A_j \cap B_j = 0_\sim$. Now consider the basic IFOSs $\prod A_k$ and $\prod B_k$ where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then $\{A(x) = (\mu_A, \nu_A)(x) = \prod A_k(x) = (\inf_{k \in J} \mu_{A_k}(x_k), \sup_{k \in J} \nu_{A_k}(x_k))\}$ and $\{B(y) = (\mu_B, \nu_B)(y) = \prod B_k(y) = (\inf_{k \in J} \mu_{B_k}(y_k), \sup_{k \in J} \nu_{B_k}(y_k))\}$. Now, $\{\mu_A(x) = \inf_{k \in J} \mu_{A_k}(x_k) = 1$, $\nu_A(x) = \sup_{k \in J} \nu_{A_k}(x_k) = 0\}$; $\{\mu_B(y) = \inf_{k \in J} \mu_{B_k}(y_k) \geq \alpha$, $\nu_B(y) = \sup_{k \in J} \nu_{B_k}(y_k) = 0\}$ and $\prod A_k \cap \prod B_k = 0_\sim$ because for $j \in J$, $A_j \cap B_j = 0_\sim$. Hence (X, t) is α-IF-T_2(i).

For n = ii, iii, it can be shown that if suppose $\{(X_m, t_m) : m \in J\}$ is a family of IFTS and (X, t) is their product IFTS. Then the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-T_2(n) if each (X_m, t_m) is α-IF-T_2(n).

4.5 Mappings in intuitionistic fuzzy T_2-spaces:

Theorem 4.5.1 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f : X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_2(i) $\Leftrightarrow$ (Y, s) is IF-T_2(i).

(2) (X, t) is IF-T_2(ii) $\Leftrightarrow$ (Y, s) is IF-T_2(ii).

(3) (X, t) is IF-T_2(iii) $\Leftrightarrow$ (Y, s) is IF-T_2(iii).

(4) (X, t) is IF-T_2(iv) $\Leftrightarrow$ (Y, s) is IF-T_2(iv).

Proof (1): Suppose the IFTS (X, t) is IF-T_2(i). We shall prove that the IFTS (Y, s) is IF-T_2(i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then $\exists$ $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since (X, t) is IF-T_2(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x_1) = 1$, $\nu_A(x_1) = 0$; $\mu_B(x_2) = 1$, $\nu_B(x_2) = 0$ and $A \cap B = 0_\sim$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1$, $(f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0\}$; $\{(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1$, $(f(\nu_B))(y_2) = \nu_B(f^{-1}(y_2)) = \nu_B(x_2) = 0\}$ and $(f(\mu_A), f(\nu_A)) \cap (f(\mu_B), f(\nu_B)) = 0_\sim$. Since f is IF-continuous, then $\{(f(\mu_A), f(\nu_A)), (f(\mu_B), f(\nu_B))\} \in s$ with $(f(\mu_A))(y_1) = 1$, $(f(\nu_A))(y_1) = 0$; $(f(\mu_B))(y_2) = 1$, $(f(\nu_B))(y_2) = 0$ and $(f(\mu_A), f(\nu_A)) \cap (f(\mu_B), f(\nu_B)) = 0_\sim$. Therefore, the IFTS (Y, s) is IF-T_2(i).

Conversely, suppose the IFTS (Y, s) is IF-T_2(i). We shall prove that the IFTS (X, t) is IF-T_2(i). Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is one-one, then $\exists$ $y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is $y_1 \neq y_2$. Since (Y, s) is IF-T_2(i), then there exists $C = (\mu_C, \nu_C)$, $D = (\mu_D, \nu_D) \in s$ such that $\mu_C(y_1) = 1$, $\nu_C(y_1) = 0$; $\mu_D(y_2) = 1$, $\nu_D(y_2) = 0$ and $C \cap D = 0_\sim$. Now, $(f^{-1}(\mu_C))(x_1) = \mu_C(f(x_1)) = \mu_C(y_1) = 1$, $(f^{-1}(\nu_C))(x_1) = \nu_C(f(x_1)) = \nu_C(y_1) = 0$; $(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1$, $(f^{-1}(\nu_D))(x_2) = \nu_D(f(x_2)) = \nu_D(y_2) = 0$ and $\{f^{-1}(\mu_C), f^{-1}(\nu_C)) \cap (f^{-1}(\mu_D), f^{-1}(\nu_D)\} = 0_\sim$. Since f is IF-continuous, then $\{(f^{-1}(\mu_C), f^{-1}(\nu_C)), (f^{-1}(\mu_D), f^{-1}(\nu_D))\} \in t$ with $(f^{-1}(\mu_C))(x_1) = 1$, $(f^{-1}(\nu_C))(x_1) = 0$;

$(f^{-1}(\mu_D))(x_2) = 1$, $(f^{-1}(\nu_D))(x_2) = 0$ and $(f^{-1}(\mu_C), f^{-1}(\nu_C)) \cap (f^{-1}(\mu_D), f^{-1}(\nu_D)) = 0_\sim$. Therefore, the IFTS (X, t) is IF-T_2(i).

(2), (3), (4) can be proved in the similar way.

Theorem 4.5.2 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(a) (X, t) is α-IF-T_2(i) $\Leftrightarrow$ (Y, s) is α-IF-T_2(i).

(b) (X, t) is α-IF-T_2(ii) $\Leftrightarrow$ (Y, s) is α-IF-T_2(ii).

(c) (X, t) is α-IF-T_2(iii) $\Leftrightarrow$ (Y, s) is α-IF-T_2(iii).

Proof (a): Suppose the IFTS (X, t) is α-IF-T_2(i). We shall prove that the IFTS (Y, s) is α-IF-T_2(i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, then $\exists$ $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. Therefore, since (X, t) is α-IF-T_2(i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x_1) = 1$, $\nu_A(x_1) = 0$; $\mu_B(x_2) \geq \alpha$, $\nu_B(x_2) = 0$ and $A \cap B = 0_\sim$. Now $\{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1$, $(f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0\}$; $\{(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) \geq \alpha$, $(f(\nu_B))(y_2) = \nu_B(f^{-1}(y_2)) = \nu_B(x_2) = 0\}$ and $(f(\mu_A), f(\nu_A)) \cap (f(\mu_B), f(\nu_B)) = 0_\sim$. Since f is IF-continuous, then $\{(f(\mu_A), f(\nu_A)), (f(\mu_B), f(\nu_B))\} \in s$ with $(f(\mu_A))(y_1) = 1$, $(f(\nu_A))(y_1) = 0$; $(f(\mu_B))(y_2) \geq \alpha$, $(f(\nu_B))(y_2) = 0$ and $(f(\mu_A), f(\nu_A)) \cap (f(\mu_B), f(\nu_B)) = 0_\sim$. Therefore, the IFTS (Y, s) is α-IF-T_2(i).

Conversely, suppose the IFTS (Y, s) is α-IF-T_2(i). We shall prove that the IFTS (X, t) is α-IF-T_2(i). Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is one-one, then $\exists\ y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is $y_1 \neq y_2$. Since (Y, s) is α-IF-T_2(i), then there exists $C = (\mu_C, \nu_C)$, $D = (\mu_D, \nu_D) \in s$ such that $\mu_C(y_1) = 1, \nu_C(y_1) = 0$; $\mu_D(y_2) \geq \alpha, \nu_D(y_2) = 0$ and $C \cap D = 0_\sim$. Now, $\{(f^{-1}(\mu_C))(x_1) = \mu_C(f(x_1)) = \mu_C(y_1) = 1$, $(f^{-1}(\nu_C))(x_1) = \nu_C(f(x_1)) = \nu_C(y_1) = 0$; $(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) \geq \alpha$, $(f^{-1}(\nu_D))(x_2) = \nu_D(f(x_2)) = \nu_D(y_2) = 0$ and $\{f^{-1}(\mu_C), f^{-1}(\nu_C))$ $\cap\ (f^{-1}(\mu_D), f^{-1}(\nu_D)\} = 0_\sim$. Since f is IF-continuous, then $\{(f^{-1}(\mu_C), f^{-1}(\nu_C))$, $(f^{-1}(\mu_D), f^{-1}(\nu_D))\} \in t$ with $(f^{-1}(\mu_C))(x_1) = 1$, $(f^{-1}(\nu_C))(x_1) = 0$; $(f^{-1}(\mu_D))(x_2) \geq \alpha$, $(f^{-1}(\nu_D))(x_2) = 0$ and $((f^{-1}(\mu_C), f^{-1}(\nu_C))$ $\cap\ (f^{-1}(\mu_D), f^{-1}(\nu_D)) = 0_\sim$.Therefore, the IFTS (X, t) is α-IF-T_2(i).

(b) and (c) can be proved in the similar way.

CHAPTER 5

On Intuitionistic Fuzzy R_0-Spaces

The concept of R_0-property was first defined by Shanin[117] and there after Dude[42], Naimpally[90], Dorsett[41], Caldas[25] and Kandil[65]. As earlier Keskin[67] and Roy[103] defined many characterizations of R_0-properties. Fuzzy R_0-properties are established by Hutton [60], Srivastava[124], Wuyts and Ali[8]. Khedr et. al.[68] also introduced the R_0-space.

In this chapter, we introduce seven notions of intuitionistic Fuzzy R_0 (in short, IF-R_0) spaces and establish some relation among them. Also, we prove that all of these definitions satisfy "good extension" property. Furthermore, we prove that all of these notions are hereditary. Finally, we observe that all these concepts are preserved under one-one, onto and continuous mappings.

5.1 Definition and Properties:

Definition 5.1.1 An intuitionistic fuzzy topological space (X, t) is called

(1) IF-R_0(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1$, $\upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) = 1$, then $\exists\ B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0$; $\mu_B(x) = 0$, $\upsilon_B(x) = 1$.

(2) IF-R_0(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) > 0$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0$; $\mu_B(x) = 0, \upsilon_B(x) > 0$.

(3) IF-R_0(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) = 1$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0, \upsilon_B(y) = 0$; $\mu_B(x) = 0, \upsilon_B(x) = 1$.

(4) IF-R_0(iv) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) > 0$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0, \upsilon_B(y) = 0$; $\mu_B(x) = 0, \upsilon_B(x) > 0$.

Definition 5.1.2 Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-R_0(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) = 1, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) \geq \alpha$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) = 1, \upsilon_B(y) = 0$; $\mu_B(x) = 0, \upsilon_B(x) \geq \alpha$.

(b) α-IF-R_0(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) \geq \alpha, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) \geq \alpha$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) \geq \alpha$, $\upsilon_B(y) = 0$; $\mu_B(x) = 0$, $\upsilon_B(x) \geq \alpha$.

(c) α-IF-R_0(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists A = (\mu_A, \upsilon_A) \in t$ with $\mu_A(x) > 0, \upsilon_A(x) = 0$; $\mu_A(y) = 0$, $\upsilon_A(y) \geq \alpha$, then $\exists B = (\mu_B, \upsilon_B) \in t$ such that $\mu_B(y) > 0$, $\upsilon_B(y) = 0$; $\mu_B(x) = 0$, $\upsilon_B(x) \geq \alpha$.

Theorem 5.1.3 The properties IF-R_0(i), IF-R_0(ii), IF-R_0(iii) and IF-R_0(iv) are all independent.

Proof: To prove the non-implications among these properties, we consider the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 1, 0), (y, 0, 0.5)} and B = {(x, 0.3, 0.2), (y, 0.1, 0.4)}. We see that the IFTS (X, t) is IF-R_0(i), but it is neither IF-R_0(ii) nor IF-R_0(iv).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.6, 0), (y, 0, 1)} and B = {(x, 0.2, 0.7), (y, 0.6, 0.3)}. We see that the IFTS (X, t) is IF-R_0(i), but it is not IF-R_0(iii).

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.5, 0), (y, 0, 1)} and B = {(x, 0.2, 0.4), (y, 0.1, 0.6)}. We see that the IFTS (X, t) is IF-R_0(ii), but it is neither IF-R_0(iii) nor IF-R_0(iv).

Example (d) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 1, 0), (y, 0, 1)} and B = {(x, 0, 0.2), (y, 1, 0)}. We see that the IFTS (X, t) is IF-R_0(ii), but it is not IF-R_0(i).

Example (e) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A = {(x, 0.6, 0), (y, 0, 0.4)} and B = {(x, 0.3, 0.1), (y, 0.4, 0.2)}. We see that the IFTS (X, t) is IF-R_0(iii), but it is not IF-R_0(iv).

Example (f) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 0.5, 0)\}$. We see that the IFTS (X, t) is IF-R_0(iii), but it is not IF-R_0(i).

Example (g) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.2, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 0.5), (y, 0.6, 0)\}$. We see that the IFTS (X, t) is IF-R_0(iv), but it is not IF-R_0(iii).

Theorem 5.1.4 The properties α-IF-R_0(i), α-IF-R_0(ii) and α-IF-R_0(iii) are all independent.

Proof: To prove the non-implications among these properties, we consider the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.6, 0), (y, 0, 0.8)\}$ and $B = \{(x, 0.4, 0.3), (y, 0.5, 0.2)\}$. For $\alpha = 0.2$, we see that the IFTS (X, t) is α-IF-R_0(i), but it is neither α-IF-R_0(ii) nor α-IF-R_0(iii).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 0.5)\}$ and $B = \{(x, 0, 0.6), (y, 0.4, 0)\}$. For $\alpha = 0.3$, we see that the IFTS (X, t) is α-IF-R_0(ii), but it is not α-IF-R_0(i).

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.5)\}$ and $B = \{(x, 0, 0.1),$

(y, 0.3, 0)}. For α =0.5, we see that the IFTS (X, t) is α-IF-R_0(ii), but it is not α-IF-R_0(iii).

Example (d) Let X = {x, y} and t be the intuitionistic fuzzy topology on X generated by {A, B} where A = {(x, 1, 0), (y, 0, 0.7)} and B ={(x, 0, 0.3), (y, 0.2, 0)}. For α =0.3, we see that the IFTS (X, t) is α-IF-R_0(iii), but it is neither α-IF-R_0(i) nor α-IF-R_0(ii).

5.2 Subspace:

Theorem 5.2.1 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$, then

(1) (X, t) is IF-R_0(i) $\Longrightarrow$ (U, t_U) is IF-R_0(i).

(2) (X, t) is IF-R_0(ii) $\Longrightarrow$ (U, t_U) is IF-R_0(ii).

(3) (X, t) is IF-R_0(iii) $\Longrightarrow$ (U, t_U) is IF-R_0(iii).

(4) (X, t) is IF-R_0(iv) $\Longrightarrow$ (U, t_U) is IF-R_0(iv).

Proof (1): Suppose (X, t) is the IFTS IF-R_0(i). We shall prove that (U, t_U) is IF-R_0(i). Let $x, y \in U$, $x \neq y$ with $A_U = (\mu_{A_U}, \nu_{A_U}) \in t_U$ such that $\mu_{A_U}(x) = 1$, $\nu_{A_U}(x) = 0$; $\mu_{A_U}(y) = 0$, $\nu_{A_U}(y) = 1$. Suppose $A = (\mu_A, \nu_A) \in t$ is the extension IFS of A_U on X, then $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$. Since $x, y \in U \subseteq X$. That is, $x, y \in X$. Again, since (X, t) is IF-R_0(i), then $\exists$ $B = (\mu_B, \nu_B) \in t$ such that $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) = 1 \Longrightarrow$ $(\mu_B|U)(y) = 1$, $(\nu_B|U)(y) = 0$; $(\mu_B|U)(x) = 0$, $(\nu_B|U)(x) = 1$. Hence $(\mu_B|U, \nu_B|U) \in t_U$. Therefore (U, t_U) is IF-R_0(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 5.2.2 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ and $\alpha \in (0, 1)$, then

(a) (X, t) is α-IF-R_0(i) $\Longrightarrow$ (U, t_U) is α-IF-R_0(i).

(b) (X, t) is α-IF-R_0(ii) $\Longrightarrow$ (U, t_U) is α-IF-R_0(ii).

(c) (X, t) is α-IF-R_0(iii) $\Longrightarrow$ (U, t_U) is α-IF-R_0(iii).

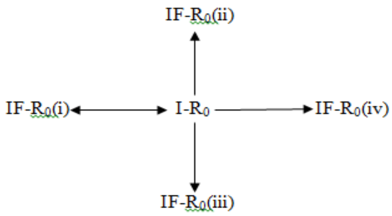
Proof (a): Suppose (X, t) is the IFTS α-IF-R_0(i). We shall prove that (U, t_U) is α-IF-R_0(i). Let $x, y \in U$, $x \neq y$ with $A_U = (\mu_{A_U}, \nu_{A_U}) \in t_U$ such that $\mu_{A_U}(x) = 1$, $\nu_{A_U}(x) = 0$; $\mu_{A_U}(y) = 0, \nu_{A_U}(y) \geq \alpha$. Suppose $A = (\mu_A, \nu_A) \in t$ is the extension IFS of A_U on X, then $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$. Since $x, y \in U \subseteq X$. That is, $x, y \in X$. Again, since (X, t) is α-IF-R_0(i), then $\exists\ B = (\mu_B, \nu_B) \in t$ such that $\mu_B(y) = 1, \nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha \Longrightarrow (\mu_B|U)(y) = 1$, $(\nu_B|U)(y) = 0$; $(\mu_B|U)(x) = 0$, $(\nu_B|U)(x) \geq \alpha$. Hence $(\mu_B|U, \nu_B|U) \in t_U$. Therefore (U, t_U) is α-IF-R_0(i).

(b) and (c) can be proved in the similar way.

5.3 Good Extension:

Definition 5.3.1 An intuitionistic topological space (X, τ) is called intuitionistic R_0-space (I-R_0 space) if for all $x, y \in X$, $x \neq y$ whenever $\exists\ C = (C_1, C_2) \in \tau$ with $x \in C_1$, $x \notin C_2$ and $y \notin C_1$, $y \in C_2$ then $\exists\ D = (D_1, D_2) \in \tau$ such that $y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $x \in D_2$.

Theorem 5.3.2 Let (X, τ) be an intuitionistic topological space and let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be I-R_0 space. We shall show that (X, t) is IF-R_0(i). Suppose (X, τ) is I-R_0. Let $x, y \in X$, $x \neq y$ with $(1_{C_1}, 1_{C_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$ and $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1 \Rightarrow x \in C_1$, $x \notin C_2$ and $y \notin C_1$, $y \in C_2$. Hence $(C_1, C_2) \in \tau$. Since (X, τ) is I-R_0, then $\exists\ (D_1, D_2) \in \tau$ such that $y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $x \in D_2 \Rightarrow 1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $1_{D_1}(x) = 0$, $1_{D_2}(x) = 1 \Rightarrow (1_{D_1}, 1_{D_2}) \in t$. Hence (X, t) is IF-R_0(i). Therefore I-$R_0 \Rightarrow$ IF-R_0(i).

Conversely, let (X, t) be IF-R_0(i). We shall show that (X, τ) is I-R_0. Suppose (X, t) is IF-R_0(i). Let $x, y \in X$, $x \neq y$ with $C = (C_1, C_2) \in \tau$ such that $x \in C_1$, $x \notin C_2$ and $y \notin C_1$, $y \in C_2 \Rightarrow 1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$ and $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1$. Hence $(1_{C_1}, 1_{C_2}) \in t$. Since (X, t) is IF-R_0(i), then $\exists$ $(1_{D_1}, 1_{D_2}) \in t$ such that $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $1_{D_1}(x) = 0$, $1_{D_2}(x) = 1$ $\Rightarrow y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $y \in D_2 \Rightarrow (D_1, D_2) \in \tau$. Hence (X, τ) is I-R_0. Hence IF-R_0(i) $\Rightarrow$ I-R_0. Therefore I-$R_0 \Leftrightarrow$ IF-R_0(i).

Furthermore, it can prove that I-$R_0 \Rightarrow$ IF-R_0(ii), I-$R_0 \Rightarrow$ IF-R_0(iii) and I-R_0 $\Rightarrow$ IF-R_0(iv).

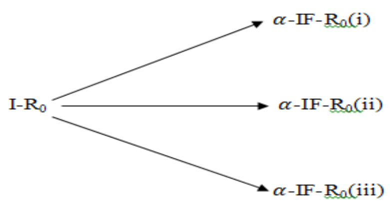
None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 0.4)\}$ and $B = \{(x, 0, 0.5), (y, 1, 0)\}$, we see that the IFTS (X, t) is IF-R_0(ii), but not corresponding I-R_0.

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.5, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 0.6, 0)\}$, we see that the IFTS (X, t) is IF-R_0(iii), but not corresponding I-R_0.

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 0.3)\}$ and $B = \{(x, 0, 0.4), (y, 0.6, 0)\}$, we see that the IFTS (X, t) is IF-R_0(iv), but not corresponding I-R_0.

Theorem 5.3.3 Let (X, τ) be an intuitionistic topological space and (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be I-R_0 space. We shall show that (X, t) is α-IF-R_0(i). Let $\alpha \in (0, 1)$. Suppose (X, τ) is I-R_0. Let $x, y \in X$, $x \neq y$ with $(1_{C_1}, 1_{C_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{C_1}(y) = 0$, $1_{C_2}(y) = 1 \Rightarrow 1_{C_1}(x) = 1$,

$1_{C_2}(x) = 0;\ 1_{C_1}(y) = 0,\ 1_{C_2}(y) \geq \alpha$ for any $\alpha \in (0,\ 1) \Rightarrow x \in C_1,\ x \notin C_2$; $y \notin C_1, y \in C_2$. Hence $(C_1,\ C_2) \in \tau$. Since $(X,\ \tau)$ is I-R_0, then $\exists\ (D_1,\ D_2) \in \tau$ such that $y \in D_1$, $y \notin D_2$ and $x \notin D_1$, $x \in D_2 \Rightarrow 1_{D_1}(y) = 1,\ 1_{D_2}(y) = 0$; $1_{D_1}(x) = 0,\ 1_{D_2}(x) = 1 \Rightarrow 1_{D_1}(y) = 1,\ 1_{D_2}(y) = 0;\ 1_{D_1}(x) = 0,\ 1_{D_2}(x) \geq \alpha$ for $\alpha \in (0,\ 1) \Rightarrow (1_{D_1},\ 1_{D_2}) \in t$. Hence $(X,\ t)$ is α-IF-R_0(i). Therefore I-R_0 $\Rightarrow$ α-IF-R_0(i).

Furthermore, it can prove that I-R_0 $\Rightarrow$ α-IF-R_0(ii) and I-R_0 $\Rightarrow$ α-IF-R_0(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x,\ y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A,\ B\}$ where $A = \{(x, 1, 0), (y, 0, 0.7)\}$ and $B = \{(x, 0, 0.8), (y, 1, 0)\}$. For $\alpha = 0.7$, we see that the IFTS $(X,\ t)$ is α-IF-R_0(i) but not corresponding I-R_0.

Example (b) Let $X = \{x,\ y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A,\ B\}$ where $A = \{(x, 1, 0), (y, 0, 0.5)\}$ and $B = \{(x, 0, 0.6), (y, 0.6, 0)\}$. For $\alpha = 0.5$, we see that the IFTS $(X,\ t)$ is α-IF-R_0(ii) but not corresponding I-R_0.

Example (c) Let $X = \{x,\ y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where $A = \{(x, 0.3, 0), (y, 0, 0.5)\}$ and $B = \{(x, 0, 0.5), (y, 1, 0)\}$. For $\alpha = 0.4$, we see that the IFTS $(X,\ t)$ is α-IFR_0(iii) but not corresponding I-R_0.

5.4 Mappings in intuitionistic fuzzy R_0-spaces:

Theorem 5.4.1 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f: X \rightarrow Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-R_0(i) $\Leftrightarrow$ (Y, s) is IF-R_0(i).

(2) (X, t) is IF-R_0(ii) $\Leftrightarrow$ (Y, s) is IF-R_0(ii).

(3) (X, t) is IF-R_0(iii) $\Leftrightarrow$ (Y, s) is IF-R_0(iii).

(4) (X, t) is IF-R_0(iv) $\Leftrightarrow$ (Y, s) is IF-R_0(iv).

Proof (1) Suppose the IFTS (X, t) is IF-R_0(i). We shall prove that the IFTS (Y, s) is IF-R_0(i). Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ with $A = (\mu_A, \nu_A) \in s$ such that $\mu_A(y_1) = 1$, $\upsilon_A(y_2) = 1$. Since f is onto, then $\exists\, x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$. Hence $x_1 \neq x_2$. We have $(f^{-1}(\mu_A), f^{-1}(\nu_A)) \in t$, as f is IF-continuous. Now, $(f^{-1}(\mu_A))(x_1) = \mu_A(f(x_1)) = \mu_A(y_1) = 1$ and $(f^{-1}(\nu_A))(x_2) = \nu_A(f(x_2)) = \nu_A(y_2) = 1$. Therefore, since (X, t) is IF-R_0(i), then $\exists\, B = (\mu_B, \nu_B) \in t$ such that $\mu_B(x_2) = 1$, $\nu_B(x_1) = 1$. Now, $(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1$ and $(f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) = 1$ as f is one-one and onto. Hence $(f(\mu_B), f(\nu_B)) \in s$. Therefore (Y, s) is IF-R_0(i). $f^{-1}(y_2)$

Conversely, suppose the IFTS (Y, s) is IF-R_0(i). We shall prove that the IFTS (X, t) is IF-R_0(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ with $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x_1) = 1$, $\upsilon_A(x_2) = 1$. Since f is one-one, then $\exists\, y_i \in s$ such that $y_i = f(x_i)$, $i = 1, 2$. Hence $f(x_1) \neq f(x_2)$ implies $y_1 \neq y_2$ as f is one-one. We have $(f(\mu_A), f(\nu_A)) \in s$ as f is IF-continuous. Now, $(f(\mu_A))(y_1) =$

$(f(\mu_A))(f(x_1)) = \mu_A(f^{-1}(f(x_1))) = \mu_A(x_1) = 1$and $(f(\nu_A))(y_2) = \nu_A(f^{-1}(f(x_2)))$ $= \nu_A(x_2) = 1$.Therefore, since (Y, s) is IF-R_0(i), then $\exists\ D = (\mu_D, \nu_D) \in s$ such that $\mu_D(y_2) = 1$, $\nu_D((y_1) = 1$. Now, $(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1$ and $(f^{-1}(\nu_D))(x_1) = \nu_D(f(x_1)) = \nu_D(y_1) = 1$, as f is one-one and onto. Hence $(f^{-1}(\mu_D), f^{-1}(\nu_D)) \in t$. Therefore is (X, t) IF-R_0(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 5.4.2 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f: X \rightarrow Y$ be one-one, onto, continuous open mapping, then

(a) (X, t) is α-IF-R_0(i) $\Leftrightarrow$ (Y, s) is α-IF-R_0(i).

(b) (X, t) is α-IF-R_0(ii) $\Leftrightarrow$ (Y, s) is α-IF-R_0(ii).

(c) (X, t) is α-IF-R_0(iii) $\Leftrightarrow$ (Y, s) is α-IF-R_0(iii).

Proof (1) Suppose the IFTS (X, t) is α-IF-R_0(i). We shall prove that the (Y, s) is α-IF-R_0(i). Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ with $A = (\mu_A, \nu_A) \in s$ such that $\mu_A(y_1) = 1$, $\upsilon_A(y_1) = 0$; $\mu_A(y_2) = 0$, $\upsilon_A(y_2) \geq \alpha$. Since f is onto, then $\exists$ $x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$. Hence $x_1 \neq x_2$. We have $(f^{-1}(\mu_A), f^{-1}(\nu_A)) \in t$, as f is IF-continuous. Now, $(f^{-1}(\mu_A))(x_1) = \mu_A(f(x_1)) = \mu_A(y_1) = 1$, $(f^{-1}(\nu_A))(x_1)$ $= \nu_A(f(x_1)) = \nu_A(y_1) = 0$; $(f^{-1}(\mu_A))(x_2) = \mu_A(f(x_2)) = \mu_A(y_2) = 0$, $(f^{-1}(\nu_A))(x_2) = \nu_A(f(x_2)) = \nu_A(y_2) \geq \alpha$. Therefore, since (X, t) is IF-R_0(i), then $\exists$ $B = (\mu_B, \nu_B) \in t$ such that $\mu_B(x_2) = 1$, $\nu_B(x_2) = 0$; $\mu_B(x_1) = 0$, $\nu_B(x_1) \geq \alpha$. Now, $(f(\mu_B))(y_2) = \mu_B(f^{-1}(y_2)) = \mu_B(x_2) = 1$, $(f(\nu_B))(y_2) =$ $\nu_B(f^{-1}(y_2)) = \nu_B(x_2) = 0$; $(f(\mu_B))(y_1) = \mu_B(f^{-1}(y_1)) = \mu_B(x_1) = 0$,

$(f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) \geq \alpha$ as f is one-one and onto. Hence $(f(\mu_B), f(\nu_B)) \in s$. Therefore (Y, s) is IF-R_0(i).

Conversely, suppose the IFTS (Y, s) is IF-R_0(i). We shall prove that the IFTS (X, t) is IF-R_0(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ with $C = (\mu_C, \nu_C) \in t$ such that $\mu_C(x_1) = 1$, $\upsilon_C(x_1) = 0$; $\mu_C(x_2) = 0$, $\upsilon_C(x_2) \geq \alpha$. Since f is one-one, then $\exists\ y_i \in s$ such that $y_i = f(x_i)$, i = 1, 2. Hence $f(x_1) \neq f(x_2)$ implies $y_1 \neq y_2$. We have $(f(\mu_C), f(\nu_C)) \in s$ as f is IF-continuous. Now, $(f(\mu_C))(y_1) = (f(\mu_C))(f(x_1)) = \mu_C(f^{-1}(f(x_1))) = \mu_C(x_1) = 1$, $(f(\nu_C))(y_1) = (f(\nu_C))(f(x_1)) = \nu_C(f^{-1}(f(x_1))) = \nu_C(x_1) = 0$; $(f(\mu_C))(y_2) = (f(\mu_C))(f(x_2)) = \mu_C(f^{-1}(f(x_2))) = \mu_C(x_2) = 0$; $(f(\nu_C))(y_2) = (f(\nu_C)(f(x_2)) = \nu_C(f^{-1}(f(x_2))) = \nu_C(x_2) \geq \alpha$. Therefore, since (Y, s) is IF-R_0(i), then $\exists\ D = (\mu_D, \nu_D) \in s$ such that $\mu_D(y_2) = 1$, $\nu_D((y_2) = 0$; $\mu_D(y_1) = 0$, $\nu_D((y_1) \geq \alpha$. Now, $(f^{-1}(\mu_D))(x_2) = \mu_D(f(x_2)) = \mu_D(y_2) = 1$, $(f^{-1}(\nu_D))(x_2) = \nu_D(f(x_2)) = \nu_D(y_2) = 0$; $(f^{-1}(\mu_D))(x_1) = \mu_D(f(x_1)) = \mu_D(y_1) = 0$, $(f^{-1}(\nu_D))(x_1) = \nu_D(f(x_1)) = \nu_D(y_1) \geq \alpha$, as f is one-one and onto. Hence $(f^{-1}(\mu_D), f^{-1}(\nu_D)) \in t$. Therefore (X, t) is IF-R_0(i).

(b) and (c) can be proved in the similar way.

CHAPTER 6

On Intuitionistic Fuzzy R_1-Spaces

The concept of R_1-property was first defined by Yang[148] and there after Murdeshwar[89], Dorset[41] and Ekici[43]. As earlier Keskin[67] and Roy [103] defined many characterizations of R_1-properties. Hutton[60], Srivastava[125], Khedr[68], Kandil[65], Hossain[56] and many other fuzzy topologists established the concepts of fuzzy R_1-properties. They introduced R_1-space, separation axioms and found their relations with other spaces.

The purpose in this chapter is to study seven new notions of R_1-property in intuitionistic fuzzy topological spaces. All these notions satisfy "good extension" property. We give several characterizations of these notions and discuss certain relationship among them. It is shown that these notions are hereditary and projective. Moreover some of their basic properties are obtained. Finally, we observe that all these concepts are preserved under one-one, onto and continuous mappings.

6.1 Definitions and Properties:

Definition 6.1.1 An intuitionistic fuzzy topological space (X, t) is called

(1) IF-R_1(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$.

(2) IF-R_1(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(3) IF-R_1(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(4) IF-R_1(iv) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

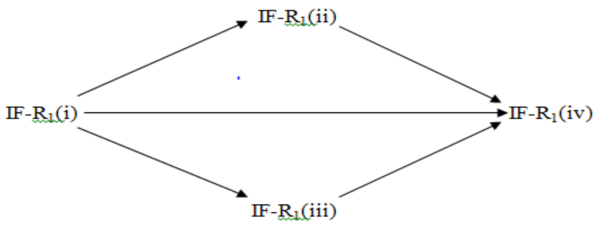
Definition 6.1.2 Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-R_1(i) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$.

(b) α-IF-R_1(ii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) \geq \alpha$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

(c) α-IF-R_1(iii) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $A = (\mu_A, \upsilon_A) \in t$ with $A(x) \neq A(y)$, then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = (0^{\sim}, \gamma^{\sim})$where $\gamma \in (0, 1]$.

Theorem 6.1.3 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implication:



Proof: Suppose (X, t) is IF-R_1(i) space. We shall prove that (X, t) is IF-R_1(ii). Let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_{\sim}$ $\Longrightarrow$ $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) > 0$, $\upsilon_C(y) = 0$ and $B \cap C = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$. Which is IF-R_1(ii). Hence IF-R_1(i) $\Longrightarrow$ IF-R_1(ii).

Again, suppose (X, t) is IF-R_1(i).We shall prove that (X, t) is IF-R_1(iii). Let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_{\sim}$ $\Longrightarrow$ $\mu_B(x) > 0$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$. Which is IF-R_1(iii). Hence IF-R_1(i) $\Longrightarrow$ IF-R_1(iii).

Furthermore, it can prove that IF-R_1(i) $\Longrightarrow$ IF-R_1(iv), IF-R_1(ii) $\Longrightarrow$ IF-R_1(iv) and IF-R_1(iii) $\Longrightarrow$ IF-R_1(iv).

None of the reverse implications is true in general which can be seen from the following examples.

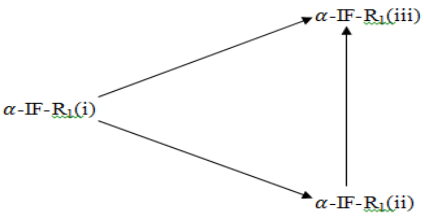
Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.2)\}$, $B = \{(x, 1, 0), (y, 0, 0.5)\}$ and $C = \{(x, 0, 0.6), (y, 0.7, 0)\}$. We see that the IFTS (X, t) is IF-R_1(ii) but not IF-R_1(i).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.3, 0), (y, 0, 0.2)\}$, $B = \{(x, 0.5, 0), (y, 0, 0.7)\}$ and $C = \{(x, 0, 0.6), (y, 1, 0)\}$. We see that the IFTS (X, t) is IF-R_1(iii) but not IF-R_1(i).

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.1, 0), (y, 0, 0.7)\}$, $B = \{(x, 1, 0), (y, 0, 0.6)\}$ and $C = \{(x, 0, 0.3), (y, 0.9, 0)\}$. We see that the IFTS (X, t) is IF-R_1(ii) but not IF-R_1(iii).

Example (d) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.4)\}$, $B = \{(x, 0.6, 0), (y, 0, 0.3)\}$ and $C = \{(x, 0, 0.5), (y, 1, 0)\}$. We see that the IFTS (X, t) is IF-R_1(iii) but not IF-R_1(ii).

Theorem 6.1.4 Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is α-IF-R_1(i) space. We shall prove that (X, t) is α-IF-R_1(ii). Let $\alpha \in (0, 1)$. Again, let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is α-IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1, \upsilon_B(x) = 0; \mu_C(y) \geq \alpha, \upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Rightarrow$ $\mu_B(x) \geq \alpha$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ for any $\alpha \in (0, 1)$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is α-IF-R_1(ii). Hence α-IF-R_1(i) $\Rightarrow$ α-IF-R_1(ii).

Again, suppose (X, t) is α-IF-R_1(ii). We shall prove that (X, t) is α-IF-R_1(iii). Let $\alpha \in (0, 1)$. Again, let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is α-IF-R_1(ii), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) \geq \alpha, \upsilon_B(x) = 0; \mu_C(y) \geq \alpha, \upsilon_C(y) = 0$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ $\Rightarrow$ $\mu_B(x) > 0, \upsilon_B(x) = 0; \mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ for $\alpha \in (0, 1)$ and $B \cap C = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is α-IF-R_1(iii). Hence α-IF-R_1(ii) $\Rightarrow$ α-IF-R_1(iii).

Furthermore, it can prove that α-IF-R_1(i) $\Rightarrow$ α-IF-R_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.2, 0), (y, 0, 0.3)}, B = {(x, 0.6, 0), (y, 0, 0.4)} and C = {(x, 0, 0.5), (y, 0.7, 0)}. For α =0.4, we see that the IFTS (X, t) is α-IF-R_1(ii) but not α-IF-R_1(i).

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.1, 0), (y, 0, 0.4)}, B = {(x, 0.3, 0), (y, 0, 0.7)} and C = {(x, 0, 0.6), (y, 0.5, 0)}. For α =0.5, we see that the IFTS (X, t) is α-IF-R_1(iii) but neither α-IF-R_1(ii) nor α-IF-R_1(i).

Theorem 6.1.5 Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then

(a) β-IF-R_1(i) $\Rightarrow$ α-IF-R_1(i).

(b) β-IF-R_1(ii) $\Rightarrow$ α-IF-R_1(ii).

(c) β-IF-R_1(iii) $\Rightarrow$ α-IF-R_1(iii).

Proof (1): Let $\beta \in (0, 1)$. Suppose the intuitionistic fuzzy topological space (X, t) is β-IF-R_1(i). We shall prove that (X, t) is α-IF-R_1(i). Let $x, y \in X$, $x \neq y$ and $A = (\mu_A, \nu_A) \in t$ with $A(x) \neq A(y)$. Since (X, t) is β-IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\nu_B(x) = 0$; $\mu_C(y) \geq \beta$, $\nu_C(y) = 0$ and $B \cap C = 0_\sim \Rightarrow \mu_B(x) = 1, \nu_B(x) = 0$; $\mu_C(y) \geq \alpha$,

$\nu_C(y) = 0$ and $B \cap C = 0_\sim$ as $0 < \alpha \leq \beta < 1$. Which is α-IF-$R_1$(i). Hence β-IF-$R_1$(i) $\Longrightarrow$ α-IF-R_1(i).

Furthermore, it can prove that β-IF-R_1(ii) ⟹ α-IF-R_1(ii) and β-IF-R_1(iii) ⟹ α-IF-R_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B, C} where A = {(x, 0.3, 0), (y, 0, 0.7)}, B = {(x, 1, 0), (y, 0, 0.4)} and C = {(x, 0, 0.5), (y, 0.6, 0)}. For α = 0.5 and β = 0.7, we see that the IFTS (X, t) is α-IF-R_1(i) but not β-IF-R_1(i).

Example (b) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B, C} where A ={(x, 0.4, 0), (y, 0, 0.2)}, B = {(x, 0.7, 0), (y, 0, 0.5)} and C = {(x, 0, 0.3), (y, 0.6, 0)}. For α = 0.6 and β = 0.8, we see that the IFTS (X, t) is α-IF-R_1(ii) but not β-IF-R_1(ii).

Example (c) Let X = {x, y} and let t be the intuitionistic fuzzy topology on X generated by {A, B, C} where A = {(x, 0.5, 0), (y, 0, 0.1), B = {(x, 0.4, 0), (y, 0, 0.6)} and C = {(x, 0, 0.3), (y, 0.5, 0)}. For α = 0.4 and β = 0.6, we see that the IFTS (X, t) is α-IF-R_1(iii) but not β-IF-R_1(iii).

6.2 Subspaces:

Theorem 6.2.1 Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ then

(1) (X, t) is IF-R_1(i) $\Rightarrow$ (U, t_U) is IF-R_1(i).

(2) (X, t) is IF-R_1(ii) $\Rightarrow$ (U, t_U) is IF-R_1(ii).

(3) (X, t) is IF-R_1(iii) $\Rightarrow$ (U, t_U) is IF-R_1(iii).

(4) (X, t) is IF-R_1(iv) $\Rightarrow$ (U, t_U) is IF-R_1(iv).

Proof (1): Suppose the IFTS (X, t) is IF-R_1(i). We shall prove that the IFTS (U, t_U) is also IF-R_1(i). Let $x, y \in U$, $x \neq y$ with $A_U = (\mu_{A_U}, \nu_{A_U}) \in t_U$ such that $A_U(x) \neq A_U(y)$. Since $x, y \in U \subseteq X$, then $x, y \in X$, $x \neq y$ as $U \subseteq X$. Suppose $A = (\mu_A, \nu_A) \in t$ is the extension IFS of A_U on X, then $A(x) \neq A(y)$. Again, since (X, t) is IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B), C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim \Rightarrow$ $(\mu_B|U)(x) = 1$, $(\nu_B|U)(x) = 0$; $(\mu_C|U)(y) = 1$, $(\nu_C|U)(y) = 0$ and $(\mu_B|U, \nu_B|U) \cap (\mu_C|U, \nu_C|U) = 0_\sim$. Hence $\{(\mu_B|U, \nu_B|U), (\mu_C|U, \nu_C|U)\} \in t_U$ $\Rightarrow (U, t_U)$ is IF-R_1(i). Therefore (U, t_U) is IF-R_1(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 6.2.2 Let $\alpha \in (0, 1)$ and let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U = \{ A|U : A \in t\}$ then

(a) (X, t) is α-IF-R_1(i) $\Rightarrow$ (U, t_U) is α-IF-R_1(i).

(b) (X, t) is α-IF-R_1(ii) $\Rightarrow$ (U, t_U) is α-IF-R_1(ii).

(c) (X, t) is α-IF-R_1(iii) $\Rightarrow$ (U, t_U) is α-IF-R_1(iii).

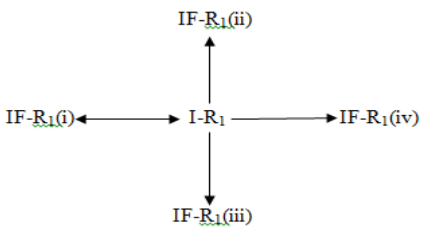
Proof (a): Suppose the IFTS (X, t) is α-IF-R_1(i). We shall prove that the IFTS (U, t_U) is also α-IF-R_1(i). Let $x, y \in U$, $x \neq y$ with $A_U = (\mu_{A_U}, \nu_{A_U}) \in t_U$ such that $A_U(x) \neq A_U(y)$. Since $x, y \in U \subseteq X$, then $x, y \in X$, $x \neq y$ as $U \subseteq X$. Suppose $A = (\mu_A, \nu_A) \in t$ is the extension IFS of A_U on X, then $A(x) \neq A(y)$. Since (X, t) is α-IF-R_1(i), then $\exists$ $B = (\mu_B, \upsilon_B)$, $C = (\mu_C, \upsilon_C) \in t$ such that $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ $\Rightarrow$ $(\mu_B|U)(x) = 1$, $(\nu_B|U)(x) = 0$; $(\mu_C|U)(y) \geq \alpha$, $(\nu_C|U)(y) = 0$ and $(\mu_B|U, \nu_B|U) \cap (\mu_C|U, \nu_C|U) = 0_\sim$. Hence $\{(\mu_B|U, \nu_B|U), (\mu_C|U, \nu_C|U)\} \in t_U$ $\Rightarrow$ (U, t_U) is α-IF-R_1(i). Therefore (U, t_U) is α-IF-R_1(i).

(b) and (c) can be proved in the similar way.

6.3 Good Extension:

Definition 6.3.1 An intuitionistic topological space (X, τ) is called intuitionistic R_1-space (I-R_1 space) if for all $x, y \in X$, $x \neq y$ whenever $\exists$ $P = (P_1, P_2) \in \tau$ with $(x \in P_1, y \in P_2)$ or $(y \in P_1, x \in P_2)$ then $\exists$ $L = (L_1, L_2)$, $M = (M_1, M_2) \in \tau$ such that $x \in L_1$, $x \notin L_2$; $y \in M_1$, $y \notin M_2$ and $L \cap M = \phi_\sim$.

Theorem 6.3.2 Let (X, τ) be an intuitionistic topological space and (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be I-R_1. We shall prove that (X, t) is IF-R_1(i). Suppose (X, τ) is I-R_1. Let $x, y \in X$, $x \neq y$ with $A = (\mu_A, \upsilon_A) \in t$ such that $A(x) \neq A(y)$. Since $A(x) \neq A(y)$, then let $(1_{C_1}(x) = 1, 1_{C_2}(y) = 1)$ or $(1_{C_1}(y) = 1, 1_{C_2}(x) = 1) \Rightarrow (x \in C_1, y \in C_2)$ or $(y \in C_1, x \in C_2)$. Hence $(C_1, C_2) \in \tau$. Since (X, τ) is I-R_1, then $\exists$ $L = (L_1, L_2)$, $M = (M_1, M_2) \in \tau$ such that $x \in L_1$, $x \notin L_2$; $y \in M_1$, $y \notin M_2$ and $L \cap M = \phi_\sim \Rightarrow 1_{L_1}(x) = 1$, $1_{L_2}(x) = 0$; $1_{M_1}(y) = 1$, $1_{M_2}(y) = 0$ and $\{(1_{L_1}, 1_{L_2}) \cap (1_{M_1}, 1_{M_2})\} = 0_\sim$. Let $\mu_B = 1_{L_1}$, $\upsilon_B = 1_{L_2}$ $\mu_C = 1_{M_1}$, $\upsilon_C = 1_{M_2}$ where $B = (\mu_B, \upsilon_B)$ and $C = (\mu_C, \upsilon_C)$. Hence $(B, C) \in t$ with $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim$. Which is IF-R_1(i).

Conversely, let (X, t) be IF-R_1(i). We shall prove that (X, τ) is I-R_1. Suppose (X, t) is IF-R_1(i). Let $x, y \in X$, $x \neq y$ with $P = (P_1, P_2) \in \tau$ such that $(x \in P_1, y \in P_2)$ or $(y \in P_1, x \in P_2)$. Now $(x \in P_1, y \in P_2)$ or $(y \in P_1, x \in P_2)$ $\Rightarrow (1_{P_1}(x) = 1, 1_{P_2}(y) = 1)$ or $(1_{P_1}(y) = 1, 1_{P_2}(x) = 1)$.Hence $(1_{P_1}, 1_{P_2}) \in t$ and $(1_{P_1}, 1_{P_2})(x) \neq (1_{P_1}, 1_{P_2})(y)$. Since (X, t) is IF-R_1(i), then $\exists$ $(1_{C_1}, 1_{C_2})$, $(1_{D_1}, 1_{D_2}) \in t$ such that $1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $\{(1_{C_1}, 1_{C_2}) \cap (1_{D_1}, 1_{D_2})\} = 0_\sim \Rightarrow x \in C_1$, $x \notin C_2$; $y \in D_1$ $y \notin D_2$ and $(C_1, C_2) \cap (D_1, D_2) = \phi_\sim$. Hence (X, τ) is I-R_1. Therefore I-$R_1 \Leftrightarrow$ IF-R_1(i).

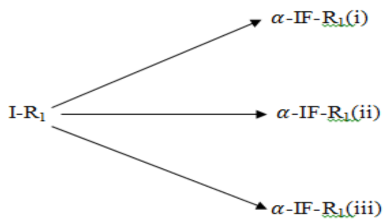
None of the reverse implications is true in general which can be seen from the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.1, 0), (y, 0, 0.5)\}$, $B = \{(x, 1, 0), (y, 0, 0.7)\}$ and $C = \{(x, 0, 0.4), (y, 0.8, 0)\}$. We see that the IFTS (X, t) is IF-R_1(ii) but not corresponding I-R_1.

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.3)\}$, $B = \{(x, 0.3, 0), (y, 0, 0.6)\}$ and $C = \{(x, 0, 0.8), (y, 1, 0)\}$. We see that the IFTS (X, t) is IF-R_1(iii) but not corresponding I-R_1.

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.3)\}$, $B = \{(x, 0.5, 0), (y, 0, 0.7)\}$ and $C = \{(x, 0, 0.5), (y, 0.9, 0)\}$. We see that the IFTS (X, t) is IF-R_1(iv) but not corresponding I-R_1.

Theorem 6.3.3 Let (X, τ) be an intuitionistic topological space and (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Let (X, τ) be I-R_1. We shall prove that (X, t) is α-IF-R_1(i). Let $\alpha \in (0, 1)$. Suppose (X, τ) is I-R_1. Let $x, y \in X$, $x \neq y$ with $A = (\mu_A, \upsilon_A) \in t$

such that $A(x) \neq A(y)$. Since $A(x) \neq A(y)$, then let $(1_{C_1}(x) = 1,\ 1_{C_2}(y) = 1)$ or $(1_{C_1}(y) = 1,\ 1_{C_2}(x) = 1) \implies (x \in C_1,\ y \in C_2)$ or $(y \in C_1,\ x \in C_2)$. Hence $(C_1, C_2) \in \tau$. Since (X, τ) is I-R_1, then $\exists\ L = (L_1, L_2)$, $M = (M_1, M_2)$ $\in \tau$ such that $x \in L_1,\ x \notin L_2$; $y \in M_1,\ y \notin M_2$ and $L \cap M = \phi_\sim \implies 1_{L_1}(x) = 1$, $1_{L_2}(x) = 0$; $1_{M_1}(y) = 1$, $1_{M_2}(y) = 0$ and $\{(1_{L_1}, 1_{L_2}) \cap (1_{M_1}, 1_{M_2})\} = 0_\sim$. Let $\mu_B = 1_{L_1}$, $\upsilon_B = 1_{L_2}$; $\mu_C = 1_{M_1}$, $\upsilon_C = 1_{M_2}$ where $B = (\mu_B, \upsilon_B)$ and $C = (\mu_C, \upsilon_C)$. Hence $(B, C) \in t$ with $\mu_B(x) = 1$, $\upsilon_B(x) = 0$; $\mu_C(y) = 1$, $\upsilon_C(y) = 0$ and $B \cap C = 0_\sim \implies \mu_B(x) = 1, \upsilon_B(x) = 0$; $\mu_C(y) \geq \alpha, \upsilon_C(y) = 0$ and $B \cap C = 0_\sim$ for any $\alpha \in (0, 1)$.Which is α-IF-R_1(i).

None of the reverse implications is true in general which can be seen by the following examples.

Example (a) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.3, 0), (y, 0, 0.4)}, B = {(x, 1, 0), (y, 0, 0.4)} and C = {(x, 0, 0.2), (y, 0.5, 0)}. For $\alpha = 0.5$, we see that the IFTS (X, t) is α-IF-R_1(i) but not corresponding I-R_1.

Example (b) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.7, 0), (y, 0, 0.9)}, B = {(x, 0.6, 0), (y, 0, 0.4)} and C = {(x, 0, 0.5), (y, 0.4, 0)}. For $\alpha = 0.3$, we see that the IFTS (X, t) is α-IF-R_1(ii) but not corresponding I-R_1.

Example (c) Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B, C\}$ where A = {(x, 0.2, 0), (y, 0, 0.6)} and B = {(x, 0.5, 0), (y, 0, 0.4)}, C ={(x, 0, 0.3), (y, 0.8, 0)}. For $\alpha = 0.7$, we see that the IFTS (X, t) is α-IF-R_1(iii) but not corresponding I-R_1.

6.4 Productivity in intuitionistic fuzzy R_1-spaces:

Theorem: 6.4.1 Let $\{(X_m, t_m) : m \in J\}$be a finite family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then each IFTS (X_m, t_m) is IF-R_1(i) if the product IFTS $(\prod X_m, \prod t_m)$ is IF-R_1(i).

Proof: Suppose the IFTS (X, t) is IF-R_1(i). We shall prove that the IFTS (X_m, t_m) is IF-R_1(i), for all $m \in J$. Let for $j \in J$, choose $x_j, y_j \in X_j$, such that $x_j \neq y_j$. Now consider $x = \prod x_m$, $y = \prod y_m$ where $x_m = y_m$ if $m \neq j$ and the jth coordinate of x, y are x_j and y_j, respectively. Then $x \neq y$. Suppose for $x_j, y_j \in X_j$, $x_j \neq y_j$ and $A_j = (\mu_{A_j}, \upsilon_{A_j}) \in t_j$ such that $A_j(x_j) \neq A_j(y_j)$. Let $A_m = (1^{\sim}, 0^{\sim})$, for $m \neq j$, then $A = \prod A_m \in t$ and $A(x) \neq A(y)$ where $A = (\mu_A, \nu_A)$. Therefore, since (X, t) is IF-R_1(i), then $\exists$ $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ such that $\mu_B(y) = 1$, $\mu_C(y) = 1$ and $B \cap C = 0_{\sim}$. Now, $\mu_B(x) = 1 \Rightarrow \inf_{m\in J} \mu_{B_m}(x_m) = 1 \Rightarrow \mu_{B_m}(x_m) = 1$ and $\mu_C(y) = 1 \Rightarrow \inf_{m\in J} \mu_{C_m}(y_m) = 1 \Rightarrow \mu_{C_m}(y_m) = 1$, for all $m \in J$. Hence we have $\mu_{B_j}(x_j) = 1$; $\mu_{C_j}(y_j) = 1$ and $B_j \cap C_j = 0_{\sim}$. Thus (X_j, t_j) is IF-R_1(i). Therefore $\{(X_m, t_m) : m \in J\}$ is IF-R_1(i).

For n = ii, iii, iv, we can prove that if suppose $\{(X_m, t_m) : m \in J\}$ is a finite family of intuitionistic fuzzy topological space and (X, t) be their product IFTS. Then each IFTS (X_m, t_m) is IF-R_1(n) if the product IFTS $(\prod X_m, \prod t_m)$ is IF-R_1(n).

Theorem: 6.4.2 Let $\{(X_m, t_m) : m \in J\}$ be a finite family of intuitionistic fuzzy topological space and (X, t) be their product. Then each IFTS (X_m, t_m) is α-IF-R_1(n) if the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-R_1(n).

Proof: Suppose the IFTS (X, t) is α-IF-R_1(i). We shall prove that the IFTS (X_m, t_m) is α-IF-R_1(i), for all $m \in J$. Let for $j \in J$, choose $x_j, y_j \in X_j$ such that $x_j \neq y_j$. Now consider $x = \prod x_m$, $y = \prod y_m$ where $x_m = y_m$ if $m \neq j$ and the jth coordinate of x, y are x_j and y_j, respectively. Then $x \neq y$. Suppose for $x_j, y_j \in X_j$, $x_j \neq y_j$ and $A_j = (\mu_{A_j}, \upsilon_{A_j}) \in t_j$ such that $A_j(x_j) \neq A_j(y_j)$. Let $A_m = (1^\sim, 0^\sim)$, for $m \neq j$, then $A = \prod A_m \in t$ and $A(x) \neq A(y)$ where $A = (\mu_A, \nu_A)$. Therefore, since (X, t) is α-IF-R_1(i), then $\exists$ $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ such that $\mu_B(x) = 1$, $\nu_B(x) = 0$; $\mu_C(y) \geq \alpha$, $\nu_C(y) = 0$ and $B \cap C = 0_\sim$. Now $\mu_B(x) = 1 \Rightarrow \inf_{m \in J} \mu_{B_m}(x_m) = 1 \Rightarrow \mu_{B_m}(x_m) = 1$, $\nu_B(x) = \sup_{m \in J} \nu_{B_m}(x_m) = 0 \Rightarrow \nu_{B_m}(x_m) = 0$; $\mu_C(y) \geq \alpha \Rightarrow \inf_{m \in J} \mu_{C_m}(y_m) \geq \alpha$ $\Rightarrow \mu_{C_m}(y_m) \geq \alpha$, $\nu_C(y) = \sup_{m \in J} \nu_{C_m}(y_m) = 0 \Rightarrow \nu_{C_m}(y_m) = 0$ for all $m \in J$. Hence we have $\mu_{B_j}(x_j) = 1$, $\nu_{B_j}(x_j) = 0$; $\mu_{C_j}(y_j) \geq \alpha$, $\nu_{C_j}(y_j) = 0$ and $B_j \cap C_j = 0_\sim$. Thus (X_j, t_j) is α-IF-R_1(i). Therefore $\{(X_m, t_m) : m \in J\}$ is α-IF-R_1(i).

For n = ii, iii, we can prove that if suppose $\{(X_m, t_m) : m \in J\}$is a finite family of intuitionistic fuzzy topological space and (X, t) is their product IFTS. Then each IFTS (X_m, t_m) is α-IF-R_1(n) if the product IFTS $(\prod X_m, \prod t_m)$ is α-IF-R_1(n).

6.5 Mappings in intuitionistic fuzzy R_1 -spaces:

Theorem 6.5.1 Let (X, t) and (Y, s) be two intuitionistic fuzzy topological spaces and $f : X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-R_1(i) $\Leftrightarrow$ (Y, s) is IF-R_1(i).

(2) (X, t) is IF-R_1(ii) $\Leftrightarrow$ (Y, s) is IF-R_1(ii).

(3) (X, t) is IF-R_1(iii) $\Leftrightarrow$ (Y, s) is IF-R_1(iii).

(4) (X, t) is IF-R_1(iv) $\Leftrightarrow$ (Y, s) is IF-R_1(iv).

Proof(1): Suppose the IFTS (X, t) is IF-R_1(i). We shall prove that the IFTS (Y, s) is IF-R_1(i). Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ and $W = (\mu_W, \nu_W) \in s$ such that $W(y_1) \neq W(y_2)$. Since f is onto, then $\exists$ $x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. We have $A = (\mu_A, \nu_A) \in t$ such that $A = f^{-1}(W)$, that is $(\mu_A, \nu_A) = (f^{-1}(\mu_W), f^{-1}(\nu_W))$ as f is IF-continuous. Now, $A(x_1) = \{\mu_A(x_1) = (f^{-1}(\mu_W))(x_1) = \mu_W(f(x_1)) = \mu_W(y_1), \nu_A(x_1) = (f^{-1}(\nu_W))(x_1) = \nu_W(f(x_1)) = \nu_W(y_1)\}$ and $A(x_2) = \{\mu_A(x_2) = (f^{-1}(\mu_W))(x_2) = \mu_W(f(x_2)) = \mu_W(y_2), \nu_A(x_2) = (f^{-1}(\nu_W))(x_2) = \nu_W(f(x_2)) = \nu_W(y_2)\}$. Hence $A(x_1) \neq A(x_2)$ as $W(y_1) \neq W(y_2)$. Therefore, since (X, t) is IF-R_1(i), then $\exists$ $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ such that $\mu_B(x_1) = 1$, $\nu_B(x_1) = 0$; $\mu_C(x_2) = 1$, $\nu_C(x_2) = 0$ and $B \cap C = 0_\sim$. Put $U = f(B)$ and $V = f(C)$ where $U = (\mu_U, \nu_U)$,

$V = (\mu_V, \nu_V) \in s$ as f is IF-continuous. Now, $\{\mu_U(y_1) = (f(\mu_B))(y_1) = \mu_B(f^{-1}(y_1)) = \mu_B(x_1) = 1, \nu_U(y_1) = (f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) = 0\}$; $\{\mu_V(y_2) = (f(\mu_C))(y_2) = \mu_C\big(f^{-1}(y_2)\big) = \mu_C(x_2) = 1, \nu_V(y_2) = (f(\nu_C))(y_2) = \nu_C(f^{-1}(y_2)) = \nu_C(x_2) = 0\}$ and $U \cap V = 0_\sim$. Hence $(U, V) \in s$. Therefore (Y, s) is IF-R_1(i).

Conversely, suppose the IFTS (Y, s) is IF-R_1(i). We shall prove that the IFTS (X, t) is IF-R_1(i). Let $x_1, x_2 \in X, x_1 \neq x_2$ and $A = (\mu_A, \nu_A) \in t$ such that $A(x_1) \neq A(x_2)$. Since f is one-one, then $\exists\ y_i \in s$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and $f(x_1) \neq f(x_2)$.That is, $y_1 \neq y_2$.We have $W = (\mu_W, \nu_W) \in s$ such that $W = f(A)$, that is $(\mu_W, \nu_W) = (f(\mu_A), f(\nu_A))$ as f is IF-continuous. Now, $W(y_1) = \{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1), f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1)\}$ and $W(y_2) = \{(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2), (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2)\}$. Hence $W(y_1) \neq W(y_2)$ as $A(x_1) \neq A(x_2)$. Since (Y, s) is IF-R_1(i), then $\exists\ U = (\mu_U, \nu_U), V = (\mu_V, \nu_V) \in s$ such that $\mu_U(y_1) = 1, \nu_U(y_1) = 0$; $\mu_V(y_2) = 1, \nu_V(y_2) = 0$ and $U \cap V = 0_\sim$. Put $B = f^{-1}(U)$ and $C = f^{-1}(V)$ where $B = (\mu_B, \nu_B), C = (\mu_C, \nu_C) \in t$ as f is IF-continuous. Now, $\{(f^{-1}(\mu_U))(x_1) = \mu_U(f(x_1)) = \mu_U(y_1) = 1, (f^{-1}(\nu_U))(x_1) = \nu_U(f(x_1)) = \nu_U(y_1) = 0\}$; $\{(f^{-1}(\mu_V))(x_2) = \mu_V(f(x_2)) = \mu_V(y_2) = 1, (f^{-1}(\nu_V))(x_2) = \nu_V(f(x_2)) = \nu_V(y_2) = 0\}$ and $B \cap C = 0_\sim$. Hence $(B, C) \in t$. Therefore (X, t) is IF-R_1(i).

(2), (3) and (4) can be proved in the similar way.

Theorem 6.5.2 Let $(X,\ t)$ and $(Y,\ s)$ be two intuitionistic fuzzy topological spaces and $f: X \to Y$ be one-one, onto, continuous open mapping, then

(a) $(X,\ t)$ is α-IF-R_1(i) $\Leftrightarrow$ $(Y,\ s)$ is α-IF-R_1(i).

(b) $(X,\ t)$ is α-IF-R_1(ii) $\Leftrightarrow$ $(Y,\ s)$ is α-IF-R_1(ii).

(c) $(X,\ t)$ is α-IF-R_1(iii) $\Leftrightarrow$ $(Y,\ s)$ is α-IF-R_1(iii).

Proof(a): Suppose the IFTS $(X,\ t)$ is α-IF-R_1(i). We shall prove that the IFTS $(Y,\ s)$ is α-IF-R_1(i). Let $y_1,\ y_2 \in Y$, $y_1 \neq y_2$ and $W = (\mu_W, \nu_W) \in s$ such that $W(y_1) \neq W(y_2)$. Since f is onto, then $\exists\ x_1,\ x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $x_1 \neq x_2$. We have $A = (\mu_A, \nu_A) \in t$ such that $A = f^{-1}(W)$, that is $(\mu_A, \nu_A) = (f^{-1}(\mu_W),\ f^{-1}(\nu_W))$ as f is IF-continuous. Now, $A(x_1) = \{\mu_A(x_1) = (f^{-1}(\mu_W))(x_1) = \mu_W(f(x_1)) = \mu_W(y_1),\quad \nu_A(x_1) = (f^{-1}(\nu_W))(x_1) = \nu_W(f(x_1)) = \nu_W(y_1)\}$ and $A(x_2) = \{\mu_A(x_2) = (f^{-1}(\mu_W))(x_2) = \mu_W(f(x_2)) = \mu_W(y_2),\ \ \nu_A(x_2) = (f^{-1}(\nu_W))(x_2) = \nu_W(f(x_2)) = \nu_W(y_2)\}$. Hence $A(x_1) \neq A(x_2)$ as $W(y_1) \neq W(y_2)$. Therefore, since $(X,\ t)$ is α-IF-R_1(i), then $\exists$ $B = (\mu_B, \nu_B),\ C = (\mu_C, \nu_C) \in t$ such that $\mu_B(x_1) = 1, \nu_B(x_1) = 0; \mu_C(x_2) \geq \alpha$, $\nu_C(x_2) = 0$ and $B \cap C = 0_{\sim}$. Put $U = f(B)$ and $V = f(C)$ where $U = (\mu_U, \nu_U)$, $V = (\mu_V, \nu_V) \in s$ as f is IF-continuous. Now, $\{\mu_U(y_1) = (f(\mu_B))(y_1) = \mu_B(f^{-1}(y_1)) = \mu_B(x_1) = 1, \nu_U(y_1) = (f(\nu_B))(y_1) = \nu_B(f^{-1}(y_1)) = \nu_B(x_1) = 0\}$; $\{\mu_V(y_2) = (f(\mu_C))(y_2) = \mu_C(f^{-1}(y_2)) = \mu_C(x_2) \geq \alpha, \nu_V(y_2) = (f(\nu_C))(y_2) = \nu_C(f^{-1}(y_2)) = \nu_C(x_2) = 0\}$ and $U \cap V = 0_{\sim}$. Hence $(U,\ V) \in s$. Therefore $(Y,\ s)$ is α-IF-R_1(i).

Conversely, suppose the IFTS (Y, s) is α-IF-R_1(i). We shall prove that the IFTS (X, t) is α-IF-R_1(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ and $A = (\mu_A, \nu_A) \in t$ such that $A(x_1) \neq A(x_2)$. Since f is one-one, then $\exists\, y_i \in s$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. We have $W = (\mu_W, \nu_W) \in s$ such that $W = f(A)$, that is $(\mu_W, \nu_W) = (f(\mu_A), f(\nu_A))$ as f is IF-continuous. Now, $W(y_1) = \{(f(\mu_A))(y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1), (f(\nu_A))(y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1)\}$ and $W(y_2) = \{(f(\mu_A))(y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2), (f(\nu_A))(y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2)\}$. Hence $W(y_1) \neq W(y_2)$ as $A(x_1) \neq A(x_2)$. Since (Y, s) is α-IF-R_1(i), then $\exists\, U = (\mu_U, \nu_U), V = (\mu_V, \nu_V) \in s$ such that $\mu_U(y_1) = 1, \nu_U(y_1) = 0$; $\mu_V(y_2) \geq \alpha$, $\nu_V(y_2) = 0$ and $U \cap V = 0_\sim$. Put $B = f^{-1}(U)$ and $C = f^{-1}(V)$ where $B = (\mu_B, \nu_B)$, $C = (\mu_C, \nu_C) \in t$ as f is IF-continuous. Now, $\{(f^{-1}(\mu_U))(x_1) = \mu_U(f(x_1)) = \mu_U(y_1) = 1, (f^{-1}(\nu_U))(x_1) = \nu_U(f(x_1)) = \nu_U(y_1) = 0\}$; $\{(f^{-1}(\mu_V))(x_2) = \mu_V(f(x_2)) = \mu_V(y_2) \geq \alpha, (f^{-1}(\nu_V))(x_2) = \nu_V(f(x_2)) = \nu_V(y_2) = 0\}$ and $B \cap C = 0_\sim$. Hence $(B, C) \in t$. Therefore (X, t) is α-IF-R_1(i).

(b) and (c) can be proved in the similar way.

References

[1]. E. Ahmed, M. S. Hossain and D. M. Ali, “On Intuitionistic Fuzzy R_0-Spaces,” *Annals of Pure and Applied Mathematics* (2015), vol. 10, no. 1, 7 –14.

[2]. E. Ahmed, M. S. Hossain and D. M. Ali, “On Intuitionistic Fuzzy R_1-Spaces,” *Journal of Mathematical and Computational Science* (2015), (Accepted).

[3]. E. Ahmed, M. S. Hossain and D. M. Ali, “Products and Mappings of Intuitionistic Fuzzy $T_{0,}$ T_1 and T_2-Spaces,” *Journal of Bangladesh Academy of Sciences* (2015), (Accepted).

[4]. E. Ahmed, M. S. Hossain and D. M. Ali, “On Intuitionistic Fuzzy T_0-Spaces,” *Journal of Bangladesh Academy of Sciences* (2014), vol. 38, no. 2, 197 –207.

[5]. E. Ahmed, M. S. Hossain and D. M. Ali, “On Intuitionistic Fuzzy T_1-Spaces,” *Journal of Physical Sciences* (2014), vol. 19, 59 –66.

[6]. E. Ahmed, M. S. Hossain and D. M. Ali, “On Intuitionistic Fuzzy T_2-Spaces,” *IOSR Journal of Mathematics* (2014), vol. 10, no. 6, 26 –30.

[7]. D. M. Ali, “On the R_1-property in Fuzzy Topology,” *Fuzzy Sets and Systems* (1992), vol. 50, 97–101.

[8]. D. M. Ali, P. Wuyts and A. K. Srivastava, “On the R_0-property in Fuzzy Topology,” *Fuzzy Sets and Systems* (1990), vol. 38, no. 1, 97 –113.

[9]. D. M. Ali, “A Note on T_0 and R_0 Fuzzy Topological Spaces,” *Proc. Math. Soc. BHU, Varanasi* (1986), 165 –167.

[10]. V. Andonov, "On Some Properties of One Certesian Product over Intuitionistic Fuzzy Sets," *Notes on Intuitionistic Fuzzy Sets* (2008), vol. 14, no. 1, 12 –19.

[11]. K. T. Atanassov, "Intuitionistic Fuzzy Sets," Theory and Applications, *Springer-Verlag* (1999), Heidelberg, New York.

[12]. K. T. Atanassov, "Review and New Results on Intuitionistic fuzzy sets," *Preprint MFAIS* (1988), vol. 88, no. 1, 1 –8.

[13]. K. T. Atanassov, "Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems* (1986), vol. 20, 87 –96.

[14]. K. T. Atanassov, "Intuitionistic Fuzzy Sets," *VII ITKR`s Session, (V, Sgurev, Ed.), Sofia* (1983), Bulgaria.

[15]. K. T. Atanassov and S. Stoeva, "Intuitionistic Fuzzy Sets," *In Polish Symposium on Interval and Fuzzy Mathematics, Poznan* (1983), 23 –26.

[16]. K. K. Azad, "On Fuzzy Pre-continuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity," *J. Math. Anal. Appl.* (1981), vol. 82, 14 –32.

[17]. K. Balachandran, P. Sundaram and H. Maki, "On Generalized Continuous Mappings in Topological Spaces," *Mem I ac. Sci. Kochi Univ. Math.* (1991), vol. 12, 5 –13.

[18]. S. Bayhan and D. Coker, "Pairwise Separation Axioms in Intuitionistic Topological Spaces," *Hacettepe Journal of Mathematics and Statistics*" (2005), vol. 34, 101 –114.

[19]. S. Bayhan and D. Coker, "On T_1 and T_2 Separation Axioms in Intuitionistic Fuzzy Topological Spaces," *Journal of Fuzzy Mathematics* (2003), vol. 11, no. 3, 581 –592.

[20]. S. Bayhan and D. Coker, "On Separation Axioms in Intuitionistic Topological Space," *Int. J. of Math. Sci*. (2001), vol. 27, no. 10, 621 – 630.

[21]. S. Bayhan and D. Coker, "On Fuzzy Separation Axioms in Intuitionistic Fuzzy Topological Space," *BUSEFAL* (1996), vol. 67, 77 –87.

[22]. A. Bhattacharjee and R. N. Bhaumik, "Pre-semi-closed Sets and Pre-semi Separation Axioms in Intuitionistic Fuzzy Topological Spaces," *Gen. Math. Notes*, (2012), vol. 8, no. 2, 11 –17.

[23]. H. Bustince, J. Kacprzyk and V. Mohedano, "Intuitionistic Fuzzy Generators: Application to Intuitionistic Fuzzy Generators Complementation," *Fuzzy Sets and Systems* (2000), vol. 114, 485–504.

[24]. H. Bustince and P. Burillo, "Structures on Intuitionistic Fuzzy Relations," *Fuzzy Sets and Systems* (1996), vol. 78, 293 –303.

[25]. M. Caldas, S. Jafari and T. Noiri, "Characterization of Pre R_0 and R_1 Topological Spaces," *Topology Proceeding* (2000), vol. 25, 17 –30.

[26]. C. L. Chang, "Fuzzy Topological Space," *J. Math. Anal. Appl*. (1968), vol. 24, 182 –190.

[27]. D. Coker, "An Introduction to Intuitionistic Topological Space," *BUSEFAL* (2000), vol. 81, 51 –56.

[28]. D. Coker, "An Introduction to Intuitionistic Topological Space," *Akdeniz University, Mathematics Department, 07058-Anatalya/ Turkey*.

[29]. D. Coker, "An Introduction to Intuitionistic Fuzzy Topological Space," *Fuzzy Sets and Systems* (1997), vol. 88, 81 –89.

[30]. D. Coker, "An Introduction to Fuzzy Subspace in Intuitionistic Fuzzy Topological Space," *J. Fuzzy Math*. (1996), vol. 4, 749 –764.

[31]. D. Coker, "A Note on Intuitionistic Sets and Intuitionistic Points," *TU J. Math*. (1996), vol. 20, no. 3, 343 –351.

[32]. D. Coker and A. Es. Hyder, "On Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces," *The Journal of Fuzzy Mathematics* (1995), vol. 3, no. 4, 899 –909.

[33]. D. Coker and M. Demirci, "On Intuitionistic Fuzzy Points," *Notes on Intuitionistic Fuzzy Sets* (1995), vol. 1, no. 2, 79−84.

[34]. E. Coskun and D. Coker, "On Neighborhood Structures in Intuitionistic Topological Spaces," *Mathematica Balkanica* (1998), vol. 12, no. 3, 289 –293.

[35]. SK. De, A. Biswas and R. Roy, "An Application of Intuitionistic Fuzzy Sets in Medical Diagnosis," *Fuzzy Sets and Systems* (2001), vol. 117, 209 –213.

[36]. SK. De, A. Biswas and R. Roy, "Some Operations on Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems* (2000), vol. 11, 477 –484.

[37]. M. Demirci, "Axiomatic Theory of Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems* (2000), vol. 110, 253–266.

[38]. G. Deschrijver and E. E. Kerre “On the Composition of Intuitionistic Fuzzy Relations,” *Fuzzy Sets and Systems* (2003), vol. 136, 333 –361.

[39]. G. Deschrijver and E. E. Kerre, “On the Relationship between Some Extensions of Fuzzy Set Theory,” *Fuzzy Sets and Systems* (2003), vol. 133, 227–235.

[40]. B. Dinda and T. K. Samanta, “Intuitionistic Fuzzy Continuity and Uniform Convergence,” *Int. J. Open Problems Compt. Math.* (2010), vol. 3, no. 1, 8 –26.

[41]. C. Dorsett, “R_0 and R_1 Topological Spaces,” *Math. Venik* (1978), vol. 2, no. 15, 117 –122.

[42]. K. K. Dude, “A Note on R_0-topological Spaces,” *Math. Vesnik* **(**1974), vol. 11, no. 36, 203 –208.

[43]. E. Ekici, “On R_1-spaces,” *International Journal of Pure and Applied Mathematics* (2005), vol. 25, no. 2, 163 –172.

[44]. M. E. El-Shafhi and A. Zhakari, “Semi Generalized Continuous Mappings in Fuzzy Topological Spaces,” *J. Egypt. Math. Soc.* (2007), vol. 15, no. 1, 57 –67.

[45]. A. H. Es and D. Coker, “More on Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces,” *Notes on Intuitionistic Fuzzy Sets* (1996), vol. 2, no. 1, 4 –10.

[46]. A. A. Fora, “Separation Axioms, Subspace and Product Spaces in Fuzzy Topology,” *Arab Culf J. Sci. Res*. (1990), vol. 8, no. 3, 1 –16.

[47]. J. Goguen, "L-fuzzy Sets," *J. Math. Anal. Applicat.* (1967), vol. 18, 145–174.

[48]. Z. Guoxu and Fu-Gui Shi, "Some Weakly Mappings on Intuitionistic Fuzzy Topological Spaces," *Tamkang Journal of Mathematics* (2008), vol. 39, no. 1, 25 –32.

[49]. H. Gurcay, D. Coker and A. Es. Hayder, "On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces," *The Journal of Mathematics of Fuzzy Mathematics* (1997), vol. 5, 365 –378.

[50]. I. M. Hanafy, "Intuitionistic Fuzzy Continuity," *Canad. Math. Bull. XX* (2009), 1 –11.

[51]. I. M. Hanafy, A. M. Abd El-Aziz and T. M. Salman, "Semi θ-continuity in Intuitionistic Fuzzy Topological Spaces," *Bull. Malays. Math. Sci. Soc.* (2006), vol. 29, 1 –10.

[52]. I. M. Hanafy, "Completely Continuous Functions in Intuitionistic Fuzzy Topological Spaces," *Czechoslovak Math*. (2003), vol. 53, no. 4, 793 –803.

[53]. M. S. Hossain and D. M. Ali, "On T_0 Fuzzy Topological Spaces," *J. Math and Math Sci.* (2009), vol. 24, 95 –102.

[54]. M. S. Hossain and D. M. Ali "On T_1 Fuzzy Bitopological Spaces," *J. Bangladesh Acad. Sci.* (2007), vol. 31, 129 –135.

[55]. M. S. Hossain and D. M. Ali, "On T_2 Fuzzy Topological Spaces," *J. Bangladesh Acad. Sci.* (2005), vol. 29, 201 –208.

[56]. M. S. Hossain and D. M. Ali, "On R_0 and R_1 Fuzzy Topological Spaces," *R. U. Studies part –B j. Sci.* (2005), vol. 33, 51–63.

[57]. M. S. Hossain and D. M. Ali, "On T_1 Fuzzy Topological Spaces," *Ganit J. Bangladesh Math. Soc.* (2004), vol. 24, 99 –106.

[58]. K. P. Huber and M. R. Berthold, "Application of Fuzzy Graphs for Metamodeling", *Proceedings of the 2002 IEEE Conference*, 640−644.

[59]. K. Hur and Y. B. Jun, "On Intuitionistic Fuzzy Alpha Continuous Mappings," *Honam Math. Jour.* (2003), vol. 25, 131 –139.

[60]. B. Hutton and I. Reilly, "Separation Axioms in Fuzzy Topological Spaces," *Fuzzy Sets and Systems* (1980), vol. 3, 93 –104.

[61]. B. Hutton, "Products of Fuzzy Topological Spaces," *J. Math. Appl.* (1980), vol. 11, 59 –67.

[62]. J. K. Jeon, Young Bae Jun and Jin Han Pak, "Intuitionistic Fuzzy Alpha Continuity and Intuitionistic Fuzzy Pre Continuity," *Int. J. of Mathematical Sci.* (2005), vol. 19, 3091 –3101.

[63]. Y. B. Jun and S. Z. Song, "Intuitionistic Fuzzy Semi-pre Open Sets and Semi-pre Continuous Mappings," *Journal of Applied Mathematics and Computing* (2005), Vol. 19, 467–474.

[64]. D. Kalamani, K. Sakthivel and C. S. Gowri, "Generalized Alpha Closed Sets in Intuitionistic Fuzzy Topological Spaces," *Applied Mathematical Sciences* (2012), vol. 6, no. 94, 4691 –4700.

[65]. A. Kandil and E. H. Sedahmed, "Fuzzy R_1-separation Axioms," *Information Sciences* (1994), vol. 79, 293 –313.

[66]. J. L. Kelley, "General Topology," *Van Nostrand, Princeton, NJ,* 1955.

[67]. A. Keskin and T. Nori, "On γ-R_0 and γ-R_1 Spaces," *Miskole Mathematics Notes* (2009), vol. 10, no. 2, 137–143.

[68]. F. H. Khedr, F. M. Zeyada and O. R. Sayed, "On Separation Axioms in Fuzzifying Topology," *Fuzzy Sets and Systems* (2001), vol. 119, 439 – 458.

[69]. Y.C. Kim and S. E. Abbas, "Separation Axioms of Intuitionistic Fuzzy Topological Spaces," *J. Egypt. Math. Soc.* (2006), vol. 14, no. 2, 353 – 375.

[70]. B. Krsteska and S. Abbas, "Intuitionistic Fuzzy Strong Pre Compactness in Coker Sense," *Mathematica Moravica* (2007), vol. 11, 59 –67.

[71]. S. J. Lee and E. P. Lee, "The Category of Intuitionistic Fuzzy Topological Spaces," *Bull. Math. Soc.* (2000), vol. 37, no. 1, 63 –76.

[72]. S. G. Li, "Separation axioms in L-fuzzy topological spaces (I): T_0 and T_1" *Fuzzy Sets and Systems* (2000), vol. 116, 377 –383.

[73]. S. Lipschutz, "General Topology," *Copyright © 1965, by the Schaum Publishing Company.*

[74]. R. Lowen, "Fuzzy Topological Spaces and Fuzzy Compactness," *J. Math. Anal. Appl.* (1976), vol. 56, 621 –633.

[75]. F. G. Lupianez, "Separation Axioms in Intuitionistic Fuzzy Topological Spaces," *International Journal of Pure and Applied Mathematics* (2004), vol. 17, 29 –34.

[76]. F. G. Lupianez. "Hausdorffness in Intuitionistic Fuzzy Topological Spaces," *Mathware and Soft Computing* (2003), vol. 10, 17 –22.

[77]. P. K. Maji, R. Biswas and A. R. Roy, "Intuitionistic Fuzzy Soft Sets," *Journal of Fuzzy Math*. (2001), vol. 9, no. 3, 677 –692.

[78]. S. R. Malghan and S. S. Benchalli, "On Open Maps, Closed Maps and Local Compactness in Fuzzy Topological Spaces," *J. Math. Anal. Appl.* (1984), vol. 99, no. 2, 338 –349.

[79]. S. R. Malghan and S. S. Benchalli, "On Fuzzy Topological Spaces," *Glasnic Mathematicki* (1981), vol. 16, no. 36, 313 –325.

[80]. A Manimaran, K. Arun Prokash and P. Thangaraj, "Intuitionistic Fuzzy Totally Continuous and Totally Semi Continuous Mappings in Intuitionistic Fuzzy Topological Space," *Int. J. Adva. Sci. Tech. Research* (2011), vol. 2, no. 1, 505 –509.

[81]. S. Maragathavalli and K. Ramesh, "On Almost π-generalized Semi Continuous Mapping in Intuitionistic Fuzzy Topological Spaces," *Mathematical Theory and Modeling* (2012), vol. 2, no. 4, 18 –28.

[82]. S. Maragathavalli and K. Ramesh, "A Note on Intuitionistic Fuzzy π-generalized Semi Irresolute Mappings," *International Journal of Mathematical Archive* (2012), vol. 3, no. 3, 1 –7.

[83]. S. Maragathavalli and K. Ramesh, "π-generalized Semi Closed Mappings in Intuitionistic Fuzzy Topological Spaces," *Journal of Advanced Studied in Topology* (2012), vol. 3, no. 4, 111 –118.

[84]. Pu. Pao. Ming and Liu Ying Ming, "Fuzzy topology I. Neighborhood Structure of a Fuzzy Point and Moore -Smith Convergence," *J. Math. Anal. Appl.* (1980), vol. 76, 571 –599.

[85]. Pu. Pao. Ming and Liu Ying Ming, "Fuzzy Topology II. Product and Quotient Spaces," *J. Math. Anal. Appl.* (1980), vol. 77, 20 –37.

[86]. K. K. Mondal, S. K. Samanta, "A Study on Intuitionistic Fuzzy Topological Spaces," *Notes on Intuitionistic Fuzzy Sets* (2003), vol. 9, 1 –32.

[87]. T. K. Mondal and S. K. Samanta, "Generalized Intuitionistic Fuzzy Sets," *Journal of Fuzzy Mathematics* (2002), vol. 10, 839 –861.

[88]. T. K. Mondal and S. K. Samanta, "On Intuitionistic Gradation of Openness," *Fuzzy Sets and Systems* (2002), vol. 131, 323 –336.

[89]. A. G. Murdeshwar and S. A. Naimpally, "R_1-topological Spaces," *Canad. Math. Bull.* (1965), vol. 9, 521 –523.

[90]. S. A. Naimpally, "On R_0-topological Spaces," *Ann. Univ. Sci. Budapest. Eotuos. Sect. Math* (1967), vol. 10, 53 –54.

[91]. AL. Narayanan, S. Vijayabalaji and N. Thillaigovindan, "Intuitionistic Fuzzy Bounded Linear Operators," *Iranian J. Fuzzy Systems* (2007), vol. 4, no. 1, 89 –101.

[92]. A. A. Nouh, "On Separation Axioms in Fuzzy Bitopological Spaces," *Fuzzy Sets and Systems* (1996), vol. 80, 225 –236.

[93]. S. Ozcag and D. Coker, "A Note on Connectedness in Intuitionistic Fuzzy Special Topological Spaces," *Int. J. Math. Math. Sciences* (2000), vol. 23, no. 1, 45 –54.

[94]. S. Ozcag and D. Coker, "On Connectedness in Intuitionistic Fuzzy Special Topological Spaces," *Int. J. Math. Math. Sciences* (1998), vol. 21, no. 1, 33 –40.

[95]. J. H. Park, "Intuitionistic Fuzzy Metric Spaces," *Chaos, Solitons Fractals* (2004), vol. 22, 1039–1046.

[96]. R. Parvathati and M. G. Karunambigai, "Intuitionistic Fuzzy Graphs, Computational Intelligence, Theory and Applications," *International Conference in Germany* (2006), 18 –20.

[97]. Z. Petricevic, "Some Separation Axioms in Fuzzy Spaces," *Zb. Rad. No.* (1990), vol. 4, 25 –33.

[98]. Z. Petricevic, "R_0 and R_1 Axioms in Fuzzy Topology," *Mat. Vesnik* (1989), vol. 41, 21 – 28.

[99]. P. Rajarajeswari and L. Senthil Kumar, "Generalized Pre-closed Sets in Intuitionistic Fuzzy Topological Spaces", *International journal of Fuzzy Mathematics and Systems* (2011), vol. 3, 253 –262.

[100]. K. Ramesh and M. Thirumalaiswamy, "Generalized Semi-pre Regular Closed Sets in Intuitionistic Fuzzy Topological Spaces," *International Journal of Computer Applications Technology and Research* (2013), vol. 2, no. 3, 324 –328.

[101]. K. Ramesh and M. Thirumalaiswamy, "Generalized Semi-pre Regular Continuous and Irresolute Mappings in Intuitionistic Fuzzy Topological Spaces," *International Journal of Engineering Research & Technology* (2013), vol. 2 no. 8, 832 –838.

[102]. S. E. Rodabaugh, “The Hausdorff Separation Axiom for Fuzzy Topological Spaces,” *Top. Appl.* (1980), vol. 11, 319 –331.

[103]. B. Roy and M. N. Mukherjee, “A Unified Theory for R_0, R_1 and Certain other Separation Properties and their Variant Form,” *Bol. Soc. Paran. Math* (2010), vol. 28, no. 2, 15 –24.

[104]. W. Rudin, “Real and Complex Analysis,” *Copyright © 1966, 1974, by McGraw –Hill Inc*.

[105]. R. Saadati and Jin Han Park, “On the Intuitionistic Fuzzy Topological Space,” *Chaos, Solitons and Fractals* (2006), vol. 27, 331 –344.

[106]. K. Sakhivel, “Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Fuzzy Alpha Generalized irresolute Mappings,” *Applied Mathematical Sciences* (2010), vol. 4, no. 37, 1831 –1842.

[107]. A. A. Salama and S. A. AL-Blowi, “Intuitionistic Fuzzy Ideals Topological Spaces,” *Journal Advanced in Fuzzy Mathematics* (2012), vol. 7, no. 1, 51 –60.

[108]. R. Santhi and K. Arun Prakash, “Intuitionistic Fuzzy Semi-generalized Irresolute Mapping,” *Tamkang Journal of Mathematics* (2011), vol. 42, 119 –134.

[109]. R. Santhi and K. Sakthivel, “Alpha Generalized Semi Homomorphism in Intuitionistic Fuzzy Topological Space,” *Notes on Intuitionistic Fuzzy Sets* (2011), vol. 17, no. 1, 30 –36.

[110]. R. Santhi and D. Jayanthi, “Intuitionisic Fuzzy Almost Generalized Semi-pre Continuous Mappings,” *Tamkang Journal of Mathematics* (2011), vol. 42, 175 –191.

[111]. R. Santhi and K. Arun Prakash, “On Intuitionistic Fuzzy Semi-generalized Closed Sets and Its Applications,” *Int. J. Contemp. Math. Sciences* (2010), vol. 5, no. 34, 1677 – 1688.

[112]. R. Santhi and K. Sakthivel, “Alpha Generalized Closed Mappings in Intuitionistic Fuzzy Topological Space,” *Far East Journal of Mathematical Sciences* (2010), vol. 43, 265 –275.

[113]. R. Santhi and D. Jayanthi, “Intuitionistic Fuzzy Generalized Semi-pre Closed Mappings,” *Notes on Intuitionistic Fuzzy Sets* (2010), vol. 3, 28 –39.

[114]. R. Santhi and K. Sakthivel, “Intuitionistic Fuzzy Generalized Semi-continuous Mappings,” *Advanced in Theoretical and Applied Mathematics* (2009), vol. 5, 73 –82.

[115]. R. Santhi and D. Jayanthi, "Intuitionistic Fuzzy Generalized Semi-pre Closed Sets," *Tripura Math. Soc.* (2009), 61 –72.

[116]. V. Seenivasan and R. Renuka, “Intuitionistic Fuzzy Pre -α-Irresolute Functions,” *Far East J. of Mathematical Sciences* (2013), vol. 72, no. 2, 251 –267.

[117]. N. A. Shanin, "On Separation in Topological Spaces," *C. R. (Doklady) Acad. Sci. URSS (N. S.)* (1943), vol. 38, 110 –113.

[118]. J. Shen, "Separation Axiom in Fuzzifying Topology," *Fuzzy Sets and Systems* (1993), vol. 57, 111 –123.

[119]. M. K. Singal and N. Prakash, "Fuzzy Pre-open Sets and Fuzzy Pre-separation Axioms," *Fuzzy Sets and Systems* (1991), vol. 44, 273 –281.

[120]. A. K. Singh and R. Srivastava, "Separation Axioms in Intuitionistic Fuzzy Topological Spaces," *Advanced in Fuzzy Systems*, November, 2012.

[121]. A. K. Singh and R. Srivastava, "On α- and α^*-separation Axioms in Intuitionistic Fuzzy Topological Spaces," *Fifteenth Int. Conf. on IFSs, Burgas* (2011), vol. no. 2, 35 –43.

[122]. S. P. Sinha, "Separation Axioms in Fuzzy Topological Spaces," *Fuzzy Sets and Systems* (1992), vol. 45, 261 –270.

[123]. R. Srivastava, S. N. Lal and A. K. Srivastava, "On Fuzzy T_1-topological Spaces," *Journal of Mathematical Analysis and Applications* (1988), vol. 46, no. 2, 124 –130.

[124]. R. Srivastava, S. N. Lal and A. K. Srivastava, "On fuzzy T_0 and R_0 Topological Spaces", *Journal of Mathematical Analysis and Applications*" (1988), vol. 136, 66 –73.

[125]. A. K. Srivastava, "R_1-fuzzy Topological Spaces," *J. Math. Anal. Appl.* (1987), vol. 127, 151 –154.

[126]. R. Srivastava and A. K. Srivastava, "On Fuzzy Hausdorff Concepts," *Fuzzy Sets and Systems* (1985), vol. 17, 67 –71.

[127]. R. Srivastava, S. N. Lal and A. K. Srivastava, "Fuzzy T_1-topological Spaces," *J. Math . Anal. Appl.* (1984), vol. 102, 4442 –4448.

[128]. R. Srivastava, S. N. Lal and A. K. Srivastava, "Fuzzy Housdorff Topological Spaces," *Journal of Mathematical Analysis and Applications* (1981), vol. 81, no. 2, 497 –506.

[129]. E. Szmidt and J. Kacprzyk, "Distances between Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems* (2000), vol. 114, 505 –518.

[130]. S. S. Thakur and Jyoti Pandey Bajpai, "On Intuitionistic Fuzzy Gpr-closed Sets," *Fuzzy Inf. Eng*. (2012), vol. 4, 425 –444.

[131]. S. S. Thakur and Bajpai Pandey Jyoti, "Intuitionistic Fuzzy w-closed Set and Intuitionistic Fuzzy w-continuity," *International Journal of Contemporary Advanced Mathematics* (2010), vol. 1, no. 1, 1 –15.

[132]. S. S. Thakur and Rekha Chaturvedi, "Generalized Closed Set in Intuitionistic Fuzzy Topology," *The Journal of Fuzzy Mathematics* (2008), vol. 16, no. 3, 559 –572.

[133]. S. S. Thakur and Rekha Chaturvedi, "Intuitionistic Fuzzy rg- continuous Mappings," *Journal of Indian Academy of Mathematics* (2007), vol. 29, no. 2, 467 –473.

[134]. S. S. Thakur and Rekha Chaturvedi, "Generalized Continuity in Intuitionistic Fuzzy Topological Spaces," *Notes on Intuitionistic Fuzzy Set* (2006), vol. 12 no. 1, 38 –44.

[135]. S. S. Thakur and Rekha Chaturvedi, “Regular Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces,” *Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Matematica* (2006), vol. 16, 257 – 272.

[136]. S. S. Thakur and Bajpai Pandey Jyoti, “Intuitionistic Fuzzy rgα-closed Sets,” *International Journal of Fuzzy Sets and Systems* (2000), vol. 116, no. 3, 369 –375.

[137]. M. Thirumalaiswamy and K. Ramesh, "Generalized Semi-pre Regular Closed Sets in Intuitionistic Fuzzy Topological Spaces," *International Journal of Computer Applications Technology and Research* (2013), Vol. 2, no. 3, 324 –328.

[138]. N. Turanh, “An Overview of Intuitionistic Fuzzy Supra Topological Spaces,” *H. J. of Math. and Stat.* (2003), vol. 32, 17 –26.

[139]. N. Turanli and D. Coker, “Fuzzy Connectedness in Intuitionistic Fuzzy Topological Spaces,” *Fuzzy Sets and Systems* (2000), vol. 116, no. 3, 369 –375.

[140]. G. J. Wang and Y. Y. He, “Intuitionistic Fuzzy Sets and L-fuzzy Sets,” *Fuzzy Sets and Systems* (2000), vol. 110, 271 –274.

[141]. M. W. Warner, “On Fuzzy Compactness,” *Fuzzy Sets and Systems* (1996), vol. 80, 15 –22.

[142]. R. H. Warren, “Fuzzy Topologies Characterized by Neighborhood Systems,” *Rocky Mountain J. Math.* (1997), vol. 9, no. 4, 761 – 764.

[143]. R. H. Warren, "Neighborhoods, Bases and Continuity in Fuzzy Topological Spaces," *Rocky Mountain J. Math.* (1978), vol. 8, 459 – 470.

[144]. C. K. Wong, "Fuzzy topology: Product and Quotient Theorem," *J. Math. Anal. Appl.* (1974), vol. 45, 512 –521.

[145]. C. K. Wong, "Fuzzy Points and Local Properties of Fuzzy Topology," *J. Math. Anal. Appl.* (1974), vol. 46, 316 –328.

[146]. P. Wuyts, "On R_1 and R_2 Fuzzy Topological Spaces, A Counter Example" *Journal of Mathematical Analysis and Applications* (1991), vol. 154, 10 –12.

[147]. P. Wuyts and R. Lowen, "On Separation Axioms in Fuzzy Topological Spaces, Fuzzy Neighborhood Spaces and Fuzzy Uniform Spaces," *J. Math. Anal. Appl*. (1973), vol. 93, 27 –41.

[148]. C. T. Yang, "On Para Compact Spaces," *Proc. Amer. Math. Soc.* (1954), vol. 5, 185 –189.

[149]. Y. L. Yue and J. M. Fang, "On Separation Axioms in I-fuzzy Topological Spaces," *Fuzzy Sets and Systems* (2006), vol. 157, 780 – 793.

[150]. L. A. Zadeh. "Fuzzy Sets," *Information and Control* (1965), vol. 8, 338 –353.